Designing of improvement efficiency of multi-channel optical communication system criteria optimization

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Abstract
One of the basic problem of the project process is to create a structure of communication systems with optimal techno-economic factors. In this paper we show optimization capital costs methods of creating the network infrastructure with reference to the defined parameters which describe a designing topology.

1. Introduction
The major problem with today’s teleinformation systems is the requirements of a very high capacity and a very high level of liveness and reliability. Unfortunately, requirements cause that costs are very high and not infrequently exceed a financial possibility of investor. Because of it, it’s necessary to take into consideration not only technical characteristics of teleinformation systems but also economical factors.

2. The optimization of network creation and exploitation costs
One of the basic problems of the project process is to create a structure of teleinformation systems with optimal techno-economic factors. We must take into consideration the cost parameters of investment besides technical factors in the project process.

Suppose that the parameters of designing teleinformation system are defined by a matrix X. Let \( \Pi(X) \) be system building and exploitation costs. The costs \( \Pi(X) \) are described as:

\[
\Pi(X) = E_n K(X) + S(X),
\]  
(1)
where: $E_n$ - a factor of investment amortization, $K(X)$ - costs for realization of investments, $\mathcal{I}(X)$ - exploitation system costs.

The investment costs $K$ are the sum of two major elements:
- cost of individual communication node,
- cost of communication channels which connect the nodes.

In order to define the costs $K$ we will present a creating network in the form of the graph $G = G(V, U, q)$, where: $V$ – the non-empty set of graph nodes, $U \subseteq \{xy\}_{x,y \in V, x \neq y}$ - the set of graph edges (on the assumption that $x$ and $y$ are nodes of graph $G$), $q$ - the function that assigns a real number $q(u) \geq 0$ which describes parameters of edges (cost, length) to each edge $u \in U$.

A number of graph nodes is described as $n = |V|$, and a number of its edges as $m = |U|$. The costs of connection nodes $x, y$ are denoted by $\chi'(K(x, y))$. The costs of creation $i$-node are denoted by $\chi''(i)$. In this case the total costs $K$ of creation network will come to:

$$K = \sum_{(x, y \in V)} \chi'(K(x, y)) + \sum_{i=1}^{n} \chi''(i).$$ (2)

When we create one-channel communication systems (in which the maximal capacity is $40 \text{Gb/s}$), the costs of creating connections dominate in the cost of creation investment. It results from a comparatively low price of devices which are intended for creation nodes. In the multi-channel communication systems the costs of connections are similar to those of their creation in the one-channel systems. But the costs of building nodal devices are much higher. For this reason, in most of multi-channel networks the building node costs dominate in the costs of a whole investment.

The capacities $P_{x,y}$ of network’s individual edges are one of the basic parameters which are placed in the matrix $X$. The excessive high capacity increases the investment cost, but if a capacity is too low, network is overloaded, and a response time is higher. The simplest way to find a relationship between the cost $\chi'$ and the capacity of connections $P_{x,y}$ is the linear dependence:

$$\chi' = \sum_{(x, y \in V)} d_{x,y} P_{x,y},$$ (3)

where: $d_{x,y}$ - a factor of building connection costs. Because lengths of individual connections are different, expression (3) doesn’t reflect a relationship between the costs of creation connection and its capacity. The better estimation can be obtained from the following dependence:

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1 We consider exploitation cost calculation at the time the system is selected.
\[ \chi' = \sum_{(x,y) \in V} d_{x,y} P_{x,y}^k, \]  

(4)

where: \( \xi \) - the factor describing the length of communication channel.

In the multi-channel systems of optical communication a capacity varies with growth of a number of virtual channels which work within the confines of one physical channel. In order to estimate the costs of creating connection we will introduce the cost function \( FK \) in the capacity function. We have following:

\[ FK(P) = \sum_{i=1}^{m} (U(P - P_i) - U(P - P_{i+1}))d_i(q), \]

(5)

where: \( m \) – the number of invariability intervals, \( P_i \) - the limit of invariability intervals of the function, \( i = 1, 2, K, m+1 \), the function \( U \) is given by:

\[ U = \begin{cases} 0, & P - P_i \leq 0 \\ 1, & P - P_i > 0 \end{cases} \]

\[ d_i(q) \] - the value of cost function in the constant sector \([P_i, P_{i+1}]\).

Taking into consideration the above models we may present expression (2) in the form that takes account of a relation between the investment cost and a capacity of edges that connect individual nodes:

\[ K = \sum_{(x,y) \in V} \chi'(FK(x,y)) + \sum_{i=1}^{n} \chi^P(i), \]

(6)

where: \( \chi^P(i) \) - the cost of creating \( i \)-node that operates the line with \( P \) capacity.

Thus in the project process we create such a structure for which:

\[ \text{opt } \Pi(X) = \min \Pi(X). \]

(7)

Because of many criteria, the optimization problem described by equation (7) is characterized by a high computational complexity. Note that the size of matrix that describes the system depends on a number of processing nodes. Therefore, when the size of system is on the increase, the solution to the optimization problem can be very difficult.

### 3. Topological methods of efficiency improvement

Topological methods to improve efficiency of operation multi-channel optical communication networks can be used in the network design stage. They reduce themselves to define the optimal number of optical network nodes, their distribution and quantity of communication channels. Notice, that both the number of optical network nodes and their distribution shouldn’t make the input data of the project process but they should be its basic result. The distribution of clients’ nodes which use optical network core resources has an influence on the optical nodes location. Next, capacity requirements of clients, their number and distribution have an influence on the optical network core capacity.
Suppose, the design system ought to serve the set of clients’ nodes $V$. Their number, i.e., power of a set $V$, were defined as $n = |V|$. The square matrix of among-nodes flows $\Lambda^{ij}$ describes intensity of communication streams which are transferred between any pair of clients’ nodes. That square matrix is:

$$\Lambda^{ij} = \begin{pmatrix} \lambda_{n1} & \text{K} & \lambda_{nn} \\ \text{M} & \text{O} & \text{M} \\ \lambda_{n1} & \text{L} & \lambda_{nn} \end{pmatrix},$$

(8)

where: $\lambda_{ij}$ - the intensity of communication stream between the nodes $i$ and $j$. Assume that communication streams which transferred between any pair of nodes are two-way, and each node can communicate with any different client’s node. Therefore, we can write down that summary intensity of communication streams $\Lambda_i$ incident to $i$ - node will be defined by a model: $\Lambda_i = \sum_{j \in V} (\lambda_{ij} + \lambda_{ji})$.

Streams incident to nodes are described by the following matrix $\Lambda'$:

$$\Lambda' = \begin{bmatrix} \Lambda_i \\ \text{M} \\ \Lambda_n \end{bmatrix},$$

(9)

The above parameters describe communication streams which will flow between the nodes of designed network. There are series of limitations connected with necessity to adjust to existing technical solutions and available transfer route of communication streams imposed on the project process. Notice, that both building costs and possible lease of communication channel costs depend on their length. For this reason, it’s necessary to take into consideration a distance between clients’ nodes in the project process. If we define a distance between the nodes $i$ and $j$ as $d_{ij}$, the distances in the system will be described by the symmetric matrix $D^{ij}$:

$$D^{ij} = \begin{pmatrix} 0 & \text{K} & d_{n1} \\ \text{M} & \text{O} & \text{M} \\ d_{n1} & \text{L} & 0 \end{pmatrix}.$$

(10)

The location of optical core nodes isn’t optional and most often connected with distribution of clients’ nodes. Let’s assume, that the set $L$ which is a subset of a set $V$ ($L \subset V$) defines acceptable core node locations. Assume, that if we distribute a core node of $r$-type network in the $k$-node, a binary digital variable $R_i^k$ must have the value of 1. Otherwise, the value of function equals 0. Moreover, let’s assume that a binary digital variable $K_{ik}$ takes the value of 1 if a client’s $i$-node is joined to core node of optical network which is distributed in a $k$-node.
Another group of design parameters are characteristics of available communication channels. The capacity of the optional communication channel is its the basic parameter. In the project process we will assume that available communication channels work at duplex mode and are symmetric. We will assume that there are \( t \) types of channels which have different capacities because optical communication channels are scalable. Let’s define the capacity of communication channels \( k \)-type as \( s_k \). In most of transmission technologies unacceptable increase in a communication channel load causes deterioration parameters of information transmission. Because of it, we can introduce constraints of using a specific communication channel in the design stage. Thus, let’s define a maximal communication channel utilization factor as \( \delta_{k \text{max}} \).

We observe similar relationships also in the case of optical network devices which are in the nodes of design network. Let’s assume, that there are \( p \) types of devices used in the system. So we can accept, that the maximal capacity of network device \( k \)-type equals \( \varphi_k \), and the cost of its purchase comes to \( \delta_k \). It’s necessary in the systems, which are weak points to design a system architecture in such a way that load of individual nodes shouldn’t exceed a definite maximal value. Moreover, let’s assume that creating network doesn’t have a homogeneous structure, but it is composed of devices which have different capacities, influence on the load of each device on functionality of the whole network is also different. So we define the maximal acceptable load of device \( r \)-type as \( \zeta_{r \text{max}} \).

The last group of project process parameters are costs connected with an execution communication channels and network core nodes. Let the \( \beta_{i,j} \) describe building costs of communication channel \( k \)-type which joins the nodes \( i \) and \( j \). Creation of the system in which all clients’ nodes are directly joined to the optical network core nodes without additional costs is impossible, that is the reason why it’s necessary to introduce a factor which describes the costs of joining a client located in \( i \)-nodes to a node distributed in the network \( k \)-point. This parameter is expressed as \( \theta_{i,j} \). The communication cost in the optical network core is an additional project process parameter. Let’s describe communication costs between the core nodes \( k \) and \( s \) as \( \Psi_{ks} (\Lambda_{ks}) \) assuming that the information stream equals \( \Lambda_{ks} \).

The project procedure which takes into consideration the above described parameters aims at minimization of the building costs and the system utilization and can be expressed as:

\[
\text{opt} \Pi(X) = \min \Pi(X) = \min \left( \sum_{i \in V} \sum_{k \in L} \theta_{ik} k + \sum_{k \in L} \sum_{r=1}^{l} \delta_{k} R_{k} + \sum_{k \in L, r \in L \wedge r > k} \Psi_{ks} (\Lambda_{ks}) \right),
\]

(11)
where: $\Lambda_{ks}$ - the summary information stream between the core node of $k$ and $s$ which is described as:

$$\Lambda_{ks} = \sum_{j \in V} \sum_{i \in V} \left( \lambda_{ij} + \lambda_{ji} \right) k_{ij} K_{js}.$$  \hspace{1cm} (12)

In order to keep correctness of the project process it’s necessary to fill up constraints referred to reciprocal connections with network nodes and essential capacities of transmission channels. Firstly, a client’s node can be joined only to one network optical core node, excessive connections are not available. Secondly, we can install devices one, defined type in any node. Moreover, in order to secure possibility of information streams switching at the full volume for each network optical core node $k$ the following condition must be satisfied:

$$\sum_{i \in V} \Lambda_{i,k} K_{ik} \leq \sum_{r=1}^{\text{max}} \zeta_r \varphi_r N_k.$$ \hspace{1cm} (13)

Creation of transmission channel costs should increase along with growth of its length. When the capacity of transmission channel is increased, then the transfer of unitary information cost ought to be decreased. Channels with the higher capacity are represented by higher numbers, and the capacity of any network optical core node is sufficient to connect no fewer network clients’ nodes than two.

4. Summary

As shown, the choice of an optimal structure of interconnection network for today’s communication systems is a very complex process. In the project process we must take into consideration technical and economical aspects.

In this paper we present expressions, which describe how to find an optimal topology. They are computationally complex.

In future, we would like to divide this complex task into smaller units. We hope, this solution will speed up the project process.

References