Some preliminary results of memory cache analysis with the use of non-extensive

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II. THEORETICAL BACKGROUND

In 1988 C. Tsallis published his famous paper [5] about the concept of new definition of non-extensive entropy. Since then a lot of papers have been published in order to critique, support, develop and discuss this proposal. The whole concept is based on a simple $q$-generalization of logarithm:

$$
\ln_q(x) = \frac{x^{1-q} - 1}{1-q}, \quad q \in R, \ln_1(x) = \ln(x) \quad (1)
$$

with inverse function given by:

$$
e^q_x = \left[ 1 + (1 - q)x \right]^{\frac{1}{1-q}} \quad (2)
$$

The equation (1) leads to the extension (or generalization) of classical definition of Boltzmann-Gibbs entropy given by

$$
S_{BG} = -k \ln(W) \quad (3)
$$

where k is Boltzmann constant and W is the number of system microstates, to the form:

$$
S_q = -k \ln_q(W) \quad (4)
$$

The development of non-extensive entropy concept introduce its continues form for the set of probabilities given by the function $p(x)$:

$$
S_q = -k \int_{-\infty}^{+\infty} p(x) \ln_q p(x) dx \quad (5)
$$
The extremization of BG entropy in continuous form

\[
S_{BG} = -k \int_{-\infty}^{+\infty} p(x) \ln p(x) \, dx \equiv_{\sigma^2\rightarrow 0} \int_{-\infty}^{+\infty} x^2 p(x) \, dx \equiv \int_{-\infty}^{+\infty} x^q p(x) \, dx
\]

with the constraints \( \int_{-\infty}^{+\infty} p(x) \, dx = 1 \), \( \int_{-\infty}^{+\infty} x \, p(x) \, dx = 0 \), \( \int_{-\infty}^{+\infty} x^2 \, p(x) \, dx = \sigma^2 \), leads to the Gaussian probability distribution given as:

\[
p_{BG}(x) = \frac{e^{-\frac{1}{2} \beta x^2}}{\sqrt{2\pi \sigma^2}},
\]

with \( \beta > 0 \) as the Lagrange parameter determined by \( \sigma^2 \). The equation (6) is considered as a ground state of the Central Limit Theorem (CLT) [6]. For non-extensive entropy (5) we can use the same way, which gives the stationary \( p_q(x) \) distribution called \( q \)-Gaussians:

\[
p_q(x) = \frac{e^{-\frac{1}{q+1} \beta x^q}}{\int_{-\infty}^{+\infty} e^{-\frac{1}{q+1} \beta x^q} \, dx}, \quad (q < 3; \beta > 0),
\]

where \( e_q \) is defined by (2). For a set of \( N \) such random where \( e_q \) is defined by (2). For a set of \( N \) such random variables their sum for \( q < 5/3 \) and \( N \rightarrow \infty \) converges to a (stable) Gaussian distribution, whereas when \( q > 5/3 \) we can obtain a set of \( \alpha \)-stable Lévy distributions [7]. Such distributions are used in statistics to generalize CLT for distributions with power laws. The \( \alpha \) (stable) parameter is described as:

\[
\alpha = \frac{3-q}{q-1} \quad \text{with} \quad q \in \left( \frac{5}{3}, 3 \right)
\]

with finite variance distributions for \( q < 5/3 \) and infinite variance distributions for \( q \geq 5/3 \).

The lack of finite variance is usually related to the existence of power-laws for analyzed system with following consequences:

- the existence of long-term and long-range dependencies;
- difficulties of calculations and interpretation of some statistical properties;
- scaling phenomena with complex dynamics;
- manifestation of non-extensive statistics.

In paper [8] it has been postulated that between the non-extensive \( q \) parameter and the Hurst exponent \( H \) exists a direct relation expressed as \( H = 1/(3-q) \).

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### III. Experiment

In order to show the complex behavior of computer memory system, a simple experiment was performed. Thanks to the perfmon tool, an useful administrator program that is included into Windows systems, we collect a set of data that are related to cache memory. Perfmon is a computer program usually used by system administrators to trace, monitor and manage different parts of computer system (Fig. 1).

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**TABLE 1** CONFIGURATIONS OF TRACED SYSTEMS AND SOME DETAILS ABOUT TIME SERIES

<table>
<thead>
<tr>
<th>C on fig</th>
<th>CPU</th>
<th>RAM</th>
<th>HDD</th>
<th>Operating system</th>
<th>Numbe r of collecte d records</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AMD Athlon 64 X2 Dual-Core TK-55, 1.80 GHz</td>
<td>3.0 GB DDR2</td>
<td>Hitachi HTS54251 2K9S400 ATA Device</td>
<td>Windows 7</td>
<td>697047</td>
</tr>
<tr>
<td>2</td>
<td>Intel Core i5 650 3.20GHz</td>
<td>4.0 GB DDR3</td>
<td>Unknown type, 500GB</td>
<td>Windows 7</td>
<td>519969</td>
</tr>
<tr>
<td>3</td>
<td>DualCore Intel Core 2 Duo T6600, 2.2 GHz</td>
<td>8.0 GB DDR3</td>
<td>ST5900325 ASG ATA Device</td>
<td>Windows 7</td>
<td>684499</td>
</tr>
<tr>
<td>4</td>
<td>Intel Pentium Dual CPU T322 2GHz</td>
<td>2.0 GB DDR2</td>
<td>Hitachi HTS54322 SL9A300 ATA Device</td>
<td>Windows 7</td>
<td>947020</td>
</tr>
<tr>
<td>5</td>
<td>Intel DualCore Celeron M T2130 2x 1.86GHz</td>
<td>2.0 GB DDR2</td>
<td>Segate ST9120822 A SATA</td>
<td>Windows 7</td>
<td>1033098</td>
</tr>
</tbody>
</table>

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Fig. 1 Perfmon main window. A new data collector set is going to be defined.

It helps to show the overview of the system and gives reports that can be used for system performance optimization. Its possibilities are based on different system counters that are normally traced by Windows system kernel, but thanks to the perfmon they also can be shown to computer user with different resolutions. This is very important, because perfmon almost doesn’t generate any additional workload (let’s note that this is normal computer program that is processed, so it must influence system performance). In experiment we trace 5 computer systems with different configurations; obtained data have at least 500k records. Details about systems configurations and traced records are presented in Table I.
The most important thing about carried experiment is fact that obtained records are based on normal work of computer user. This means that during experiment we don’t use any benchmarks or computer programs that generate any artificial workload; records represent typical work of different computer users who use office programs, internet browsers, mail clients, games, multimedia, etc. The weakness of this approach is a fact that there is no possibility to repeat unique behavior of each user, but on the other hand, in the case of personal computers they usually don’t process the extreme workload that is generated by benchmarks. Our approach shows computer system behavior during man-machine interaction. Obtained data were collected with resolution 1 s (the highest possible in perfmon) thus during 1 hour we were able to have 3.6k observations. Taking into account data presented in Table I it is clear that we trace our systems at least for 130 hours (or more). In our experiment we focused on one system counter, which helps evaluating memory and especially cache usage: Cache Bytes. It shows the amount of resident pages allocated in RAM that the Kernel threads can address without causing a page fault [9] consisting of:

- Memory\System Cache Resident Bytes as the size (in bytes) of the pageable operating system code in the file system cache (includes only current physical pages and does not include any virtual memory pages not currently resident),
- Memory\System Driver Resident Bytes as the size (in bytes) of pageable physical memory that is used by different device drivers,
- Memory\System Code Resident Bytes as the size (in bytes) of operating system code located in physical memory (it can be written to disk when is not used),
- Memory\Pool Paged Resident Bytes as the size (in bytes) of the paged pool. Space used by the paged and nonpaged pools is taken from physical memory, so a pool that is too large denies memory space to processes.

IV. RESULTS OF EXPERIMENT

Figures 1a-e show behavior of counter Cache Bytes for 5 different computer configurations. The first interesting, very well-visible fact is the behavior of this counter represented by high jumps (up and down) in cache memory usage. These jumps are related to the fact that during processing computer user runs different computer programs that require memory (especially visible by Pool Paged Resident Bytes).

Because we cannot be sure whether these time series are stationary or not (especially in Fig. 2c, where we have well-visible increasing trend), the simplest way is to analyze their increments - figures 3a-e show details.

Increments presented in Fig. 3 seems to be typical for processes that usually are described by Lévy \(\alpha\)-stable processes [10], which we described in Section 2. Figs 4a-e) show probability distributions of increments presented in Figs. 3 – each of them is done in log-lin scale with the fit of normal distribution in order to show the behavior of distribution tails.
From Figs. 4a-e) it is clear that each probability distribution (open squares) has slowly vanishing tail in comparison to normal distribution (open triangles). Figs. 5a-e) show the right tail of each distribution with linear fit that exhibits the possible existence of power laws (i.e., heavy tails).

In all cases we have results that indicate the possible existence of power-law behavior for tails distribution. The estimated slopes of fitted data are respectively: $b = -1.905$, $b = -1.761$, $b = -2.007$, $b = -2.1$, $b = -2.144$. Because we use probability distribution function, the estimation of tail values in log-log scale can be done in relation to the Eq. (10):

$$\lim_{|x|\to \infty} p(x) \approx \frac{ae^{\alpha}}{|x|^{1+\alpha}} \approx |x|^{-1-\alpha},$$  \hspace{1cm} (10)

where $C$ is a positive constant, however this is a very rough approximation giving us stability index $\alpha = -b+1$: $\alpha = 0.905$, $\alpha = 0.761$, $\alpha = 1.007$, $\alpha = 1.1$, $\alpha = 1.144$. But this method works satisfactorily only when we have ideal (simulated, not skewed) probability distributions. Here, in order to calculate the stability index we need to focus on different approach based on relation:

$$\lim_{|x|\to \infty} (1 - F(x)) \approx \frac{ae^{\alpha}}{|x|^{1+\alpha}} \approx |x|^{-\alpha},$$  \hspace{1cm} (11)

where $F(x)$ is cumulative distribution function. We obtain respectively: $\alpha = 1.825$, $\alpha = 1.651$, $\alpha = 1.781$, $\alpha = 1.93$, $\alpha = 1.69$, however again basing on graphical approach where we plot $1-F(x)$ vs $x$ on a log-log scale. In order to obtain more accurate results we need to use methods based on Hill estimator or maximum likelihood estimator.

V. CONCLUSIONS

In this paper we show behavior of one system memory counter (obtained from five different computer systems). Despite the fact, that our findings (power law behavior in analyzed time series) were done basing on graphical methods and without any parametrical statistical tests, we were able to discover some interesting features of Cache Bytes counter. Obtained results make a strong connection between Tsallis entropy, its thermodynamical background and computers. It seems that the existence of Lévy stable processes in the case of computer systems are not widely known in the case of computer systems analysis (behavior of different system parts), but on the other hand they are showing new interesting experiment possibilities. The so far obtained results need to be developed relating them to the more detailed study of time series statistical properties (for example towards statistical self-similarity in time domain [11]) and to be compared with results for other different hardware systems configurations. This task is not the easy one, however proposed approach seems to confirm that computer systems work in states that are far from equilibrium states that can be described and analyzed by the concept of Tsallis entropy. Let’s note that this thermodynamical background, which at first glance seems to be proper for considerations of combustion engines, is firmly seated in computer systems, which in fact are also machines that perform (electrical) energy transformations into useful work (calculations).

REFERENCES


