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Annales UMCS Informatica AI 2 (2004) 201-208

Annales UMCS
Informatica
Lublin-Polonia
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Histogram analysis of the human brain MR images based on the S-function membership and Shannon's entropy function

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Abstract

The analysis of medical images for the purpose of computer-aided diagnosis and therapy planning includes segmentation as a preliminary stage for the visualization or quantification. In this paper, we present the first step in our fuzzy segmentation system that is capable of segmenting magnetic resonance (MR) images of a human brain. The histogram analysis based on the S-function membership and the Shannon's entropy function provides finding exact segmentation points. In the final stage, pixel classification is performed using the rule-based fuzzy logic inference. When the segmentation is complete, attributes of these classes may be determined (e.g., volumes), or the classes may be visualized as spatial objects. In contrast to other segmentation methods, like thresholding and region-based algorithms, our methods proceeds automatically and allow more exact delineation of the anatomical structures.

1. Introduction

The computer image can be divided into the homogeneous regions that obey the given criterion. Such partition is called segmentation. The threshold techniques, which make decisions based on the local pixel information is the simplest method of segmentation. The following equation describes the image that can be divided into *k+1* homogenous regions:

$$g(k:x,y)=\begin{cases} k, & if \quad f(x,y)>T(k) \\ k-1, & T(k-1)<f(x,y)\le T(k) \\ \vdots & \\ 1, & T(0)<f(x,y)\le T(1) \\ 0, & otherwise \end{cases} \tag{1}$$

T(k) is the threshold level for *k* region, *f* returns the intensity of the pixel in *(x, y)* position in the input image and *g* is the output image. In this paper we

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present how to find segmentation points *k* for human brain MR images. For that purposes we have implemented a slightly modified version of fuzzy entropic auto thresholding algorithm based on the S-function membership and the Shannon's entropy function proposed by Fleury M., Hayat L., Clark A.F. [1].

2. Methodology

The information theoretical entropy, which measures the mean value of the uncertainty, is defined as [2]:

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i \quad H[0,1] \tag{2}$$

H is the average information supplied by a set of *i* symbols whose probabilities are given by $p_1, p_2, p_3, \ldots, p_i, \ldots, p_n$. This has the principal properties that, with the uncertainty defined as $-log_2 p$, events of lower probability are more uncertain and the uncertainty of two simultaneous events is the sum of their individual uncertainties.

The fuzzy membership is regarded as a membership graduation of a set with the statistical uncertainty. This is the exact situation where the image histogram does not display a smooth set of valleys between peaks. In our approach the standard *S*-function [3] has been selected for the purpose of the membership graduation. Its generalized form is defined as follows:

$$\mu(x) = S(x,a,b,c) = \begin{cases} 0, & x \le a \\ k\left(\dfrac{x-a}{c-a}\right)^2, & a \le x \le b, k = \dfrac{c-a}{b-a} \\ 1-k\left(\dfrac{x-c}{c-a}\right)^2, & b \le x \le c, k = \dfrac{c-a}{c-a} \\ 1, & x \ge c \end{cases} \tag{3}$$

where *a, b* and *c* are the parameters which determine the shape of the S-function (see Figure 1).

This function is symmetrical only when the cross-over point *b* lies exactly in the middle of the fuzzy region *(a, c)* (i.e. $b=(a+c)/2$, bandwidth $\Delta b=b-a=c-b$ and $k=2$).

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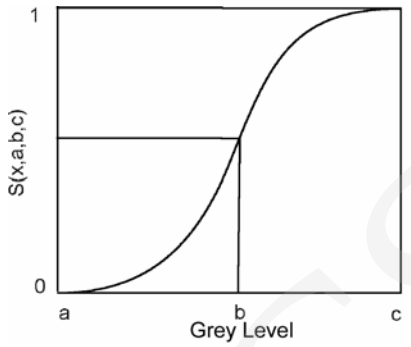

Fig. 1. Graph of the standard S-function, $b=(a+c)/2$, bandwidth $\Delta b=b-a=c-b$ and $k=2$

In comparison with a two-tone image, a gray-level image appears fuzzy. We need to define a measure of the fuzziness to enable choosing segmentation levels (boundary threshold values describing a specified region) that will minimize the fuzziness. It is possible to apply Shannon's function (equation (4), which is equation (2) with $n=2$) to the membership function for a particular bandwidth (see Figure 2):

$$S_n(x) = -\mu(x)\log_2 \mu(x) - (1-\mu(x))\log_2 (1-\mu(x)), \quad 0 < x < n-1. \qquad (4)$$

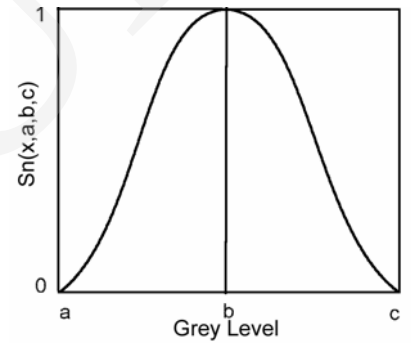

Fig. 2. Graph of the S_n function

Now the fuzzy entropy measure is given by [1]:

$$H_{fuzzy}(A) = \frac{1}{n \ln 2} \sum_{i=1}^{n} Sn(\mu_A(x_i)) \qquad (5)$$

where A is the fuzzy set of concern containing n members. To apply this equation to an image histogram H, with n grey-levels within the fuzzy region g_i and width h_i pixels in the ith histogram bin, we use (see Figure 3):

$$H_{fuzzy}(H) = \frac{1}{n \ln 2} \sum_{i=1}^{n} Sn(\mu_H(g_i))h_i \qquad (6)$$

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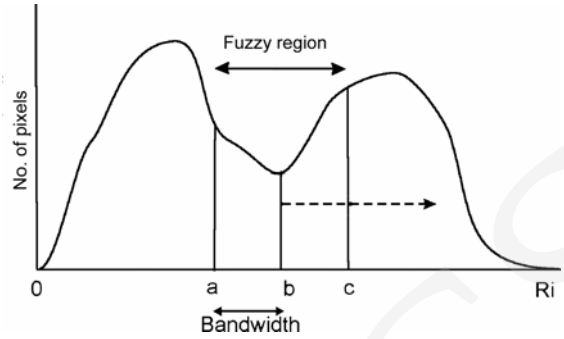

Fig. 3. Applying fuzzy region on the image histogram

The result of applying this measure to a grey-level probability distribution is illustrated in Figure 4.

This fuzzy entropy method will find a number of valleys (i.e. histogram local minima) depending on bandwidth. However, we first restricted the bandwidth range by consideration of the known number of peaks on MR Image histogram of human brain. To detect the fuzzy entropy valleys (histogram local minima) we need to find only where $e(k-1) < e(k) < e(k+1)$ for successive discretely sampled values of fuzzy entropy $e(k)$ corresponding to each starting position of the bandwidth window. That process is iteratively repeated until minima for a proper number of tissue classes are found (histogram minima are the tissue classes boundaries). Additional boundary threshold values are detected in high gradients and in 5% histogram height.

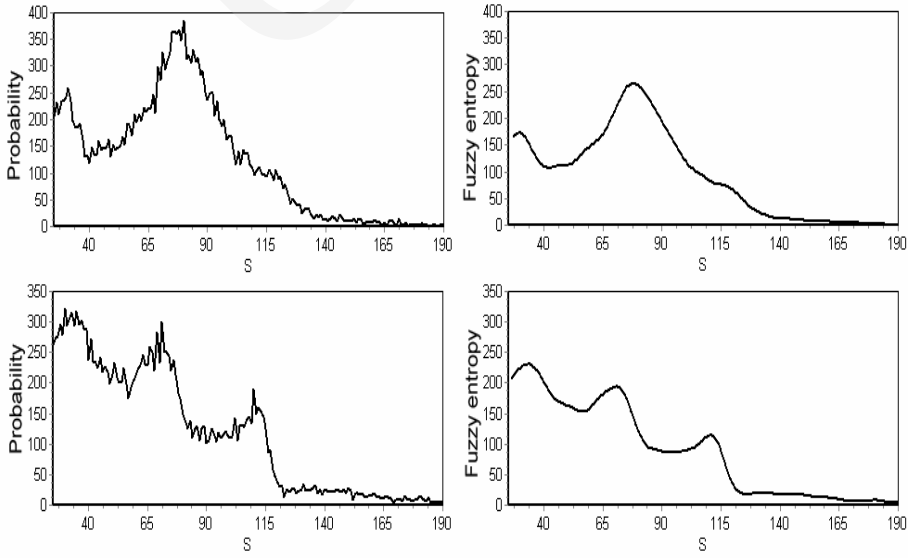

Fig. 4. Probability distribution for two exemplary MR images (left) and their fuzzy entropy functions (right)

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3. Results

We have tested the presented method to segment the MRI data sets of a human head. Figure 5 presents the histogram and entropy histograms for various bandwidths for one of MRI scan of a human head. Table 1 presents the number of detected minima and the positions of minima for each bandwidth. From the general anatomical knowledge of the brain and their MR images we have restricted the number of tissue classes to four, and the algorithm has set up proper bandwidth and has found appropriate positions of minima. Before we use this algorithm to find these minima we create the histogram of the entire data set (i.e. set of MR scans of the human brain) rather than for each single scan. Only after that we apply this algorithm to find minima. Figure 6 presents thresholded scan generated on the basis of data obtained from the fuzzy-entropic algorithm.

This segmentation is implemented as a threshold only to show that the fuzzy-entropic algorithm detects the histogram minima properly. Of course the threshold technique is not a sufficient as a segmentation method, because it leaves too many pixels not belonging to an appropriate class. For that reason this algorithm is only a part of our human brain segmentation system. Precisely, the presented fuzzy entropy histogram filter is a second step in our segmentation system. Results from this step are the entry data for the next step, pixel classification. The full system consists of four stages and is capable of complete segmenting of simple images [5]. The results of full segmentation as the three-dimensional visualization of the virtual anatomical model created on the basis of segmented structures are presented in Figure 7.

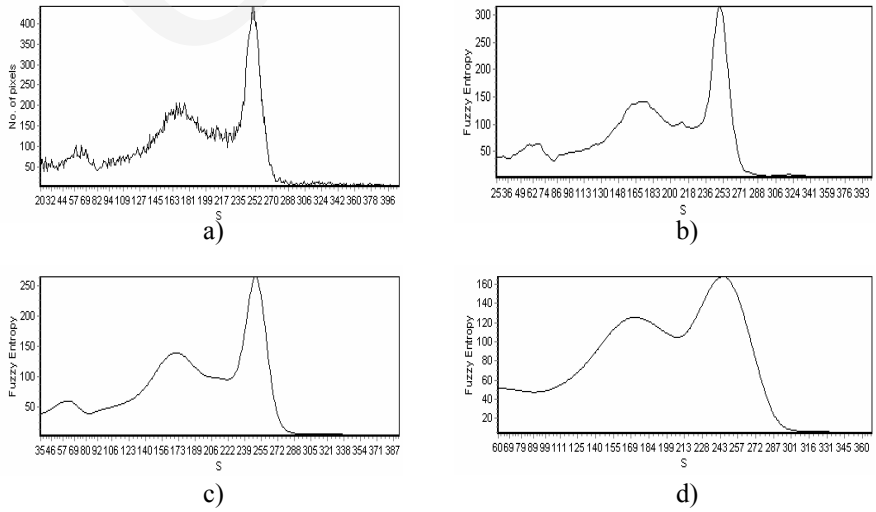

Fig. 5. The fuzzy entropy histogram of one of MR slices for various bandwidths: the original histogram (a), the fuzzy histogram with the bandwidth $\Delta b=5$ (b), $\Delta b=15$ (c), $\Delta b=40$ (d)

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Table 1. Positions of minimum entropy and the number of regions depending on bandwidth for the histogram presented in Figure 5(a)

Bandwith	No. of minima	Positions of minima
$\Delta b = 5$	13	27,37,61, 82, 93, 101, 109, 127, 163, 172, 202, 222, 273
$\Delta b = 15$	4	34, 83, 221, 276
$\Delta b = 40$	3	90, 208, 288

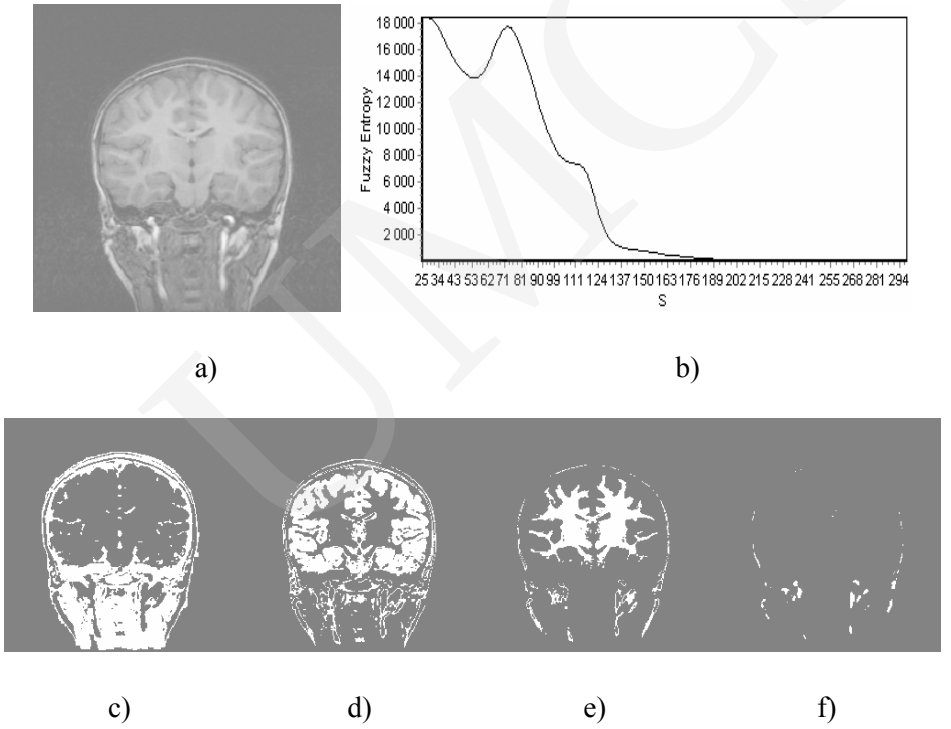

Fig. 6. Four classes segmented from MRI scan (a), fuzzy entropy histogram of the entire data set (b), classes for boundaries at: 1-53 (c), 54-93 (d), 94-131 (e), 131-445 (f).

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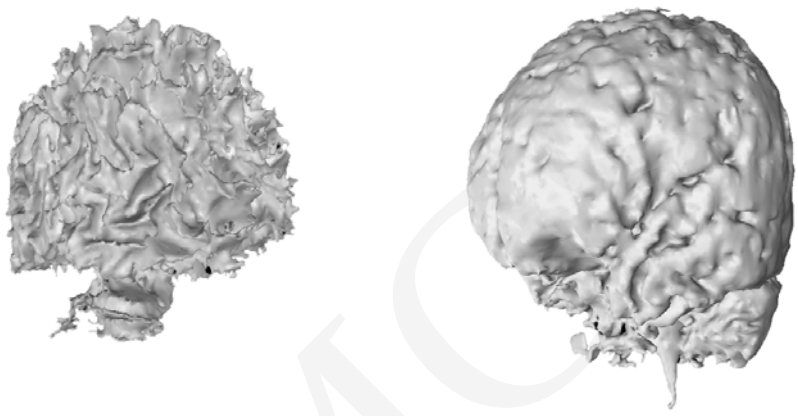

a) b)

Fig. 7. The three-dimensional visualization of segmented anatomical structures. The white matter (a), the grey matter (b)

4. Conclusions and future work

We have examined the idea of using the fuzzy entropy system as the basics for the human head MR image segmentation algorithm. In this algorithm, for a given bandwidth, image grey levels are translated into fuzzy outputs after applying a non linear transformation. The entropy of the fuzzy outputs is computed through Shannon's function [2]. The locations of valleys in the fuzzy entropy are taken as threshold points. This entropic method is computationally fast, which is important in real-time systems, and is not restricted to filtering MR human brain image histograms only.

There are many alternatives for the histogram analysis algorithm step of described algorithm. One of them was presented in our previous work [4]: semi-automatic quantitative histogram analysis. This method is based on the idea that for a large number of samples in volumetric dataset the distribution of its values for a given material will be gaussian. Under this assumption we can model the histogram as a sum of parameterized gaussian functions that make the model agree with the histogram. The image quantification is only the problem of fitting a model to the histogram data by estimating the model parameters (in this approach we have selected iterative Levenberg-Marquardt as an optimization algorithm). The main drawback of this method is that the model has to be clearly

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defined – numbers of gaussian and starting parameters have to be determined manually.

Currently we are working on combining the gaussian-based method with the one presented in this paper. Our future aim is to develop the fully automatic segmentation method with an automatic gaussian model definition step (based on the S-function membership and Shannon's entropy).

Acknowledgments

We are very grateful to the National Library of Medicine [6,7] for the permission to work on and publish the Visible-Human-male datasets.

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