A hybrid approach to segmentation of medical images

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Abstract
The paper presents information theory based image segmentation algorithms. Both advantages and problems related to them are discussed. Then a segmentation framework based on combination of information theory and fuzzy logic is proposed.

1. Introduction
Segmentation is one of the essential image processing techniques. There is a great number of known algorithms and new ones are still being created. However, in the case of medical images it is still not trivial to obtain automatically results comparable to manual segmentation performed by a human expert. In this paper we discuss some aspects and problems of information theory based (entropic) segmentation and propose a hybrid approach making use of fuzzy logic methods.

2. Medical imaging
In this chapter properties of medical digital images that are related to a segmentation task are presented. Segmentation is a process of partitioning an image into a set of homogenous regions, with respect to a given criterion. In the case of medical images these regions are expected to correspond to various tissues and pathologies to be visualized.

An image is composed of a number of voxels, that are organized in slices, usually in one of three planes: axial, sagittal or coronal (Fig. 1). Voxel intensities in anatomical modalities are proportional to various physical properties of the underlying tissues. In the case of CT (computed tomography) it is X-rays radiation absorption. In MRI (magnetic resonance imaging) these are relaxation times and proton densities, which are more difficult to interpret in the context of

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tissues. The other group – functional modalities provide information on the metabolism of the underlying anatomy rather than tissue properties and are hardly useful without anatomical modality images of the same object.

![Image of CT examination planes](image)

**Fig. 1.** Axial, sagittal and coronal planes in a CT examination

The nature of medical image data causes some serious problems in a segmentation task. Images tend to be noisy. However, their quality is still being improved. In many cases voxel intensities correspond to various tissues overlap, so that segmentation based on intensities only fails. Close to boundaries between tissues it happens, that volume represented in an image as one voxel consists of more than one kind of tissue. Then the intensity represents a superposition of signals from various tissues (partial volume effect). Because of that it is difficult to localize the exact borders. Image properties also change due to many factors, like radiation and chemotherapy treatment, which is common in the case of patients with brain tumours.

### 3. Information theory based image segmentation

There is a great number of available segmentation methods (histogram shape-based, entropy-based, clustering-based, object attribute-based, etc.) and new ones are still created. A great review can be found in [1]. Pixel (voxel) intensity thresholding is the simplest (and the most commonly applied) approach. Individual pixels are classified as “black” ($b_0$) or “white” ($b_1$) as follows [2]:

$$
j(r,c) = \begin{cases} 
  b_0, & \text{if } i(r,c) \leq t, \\
  b_1, & \text{if } i(r,c) > t.
\end{cases}
$$

(1)

where $i(r,c)$ is the grey-scale image, $j(r,c)$ is the resulting binary image, and $t$ is the threshold to be found (either globally – for the whole image, or locally for every sub-area). Multi-class segmentation can be performed by iterative segmentation of previously obtained classes.
The threshold can be found in numerous ways. Image theory provides potentially interesting methods that can be applied to image segmentation. Entropy is the crucial term in this approach. When calculated for a random variable, it is a measure of its randomness. It can be also interpreted as an information measure [3]. The most common definition is that of Shannon [4]. The entropy $H$ of a discrete random variable $X$ with values in the set \{x_1,x_2,.....x_n\} is defined as:

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i,$$

where $p_i = \text{Pr}[X=x_i]$.

The image entropy (eq. 2.) is usually estimated using a histogram [2]:

$$p_i \equiv \frac{g_i}{g_{total}},$$

where $g_i$ is the number of pixels with the intensity $i$ and $g_{total}$ is the total number of pixels, while $n$ (in eq. 2.) is the number of grey-levels. However, this approach has a significant drawback: the pixels are assumed to be independent and the spatial information is ignored. Random rearrangement of any number of pixels does not affect the histogram, so the entropy remains unchanged, too (while intuitively we would expect it to grow).

![Fig. 2. The “monkey model” of a 1-dimensional image [5]](image_url)

Another approach, often referred as a “monkey model”, assumes an image to be made up of a fixed number of photons (unit grey-levels) $G$ randomly allocated in $n$ cells (pixels, Fig. 2) [5]. Here eq. 2. is used over again for entropy calculation, but:

$$p_i = \frac{g_i}{G}$$

where $g_i$ is the grey-level of the $i$-th pixel.

In this case the spatial information is still ignored, like in the previous model. However, we can use a modified form of entropy definition (eq. 2.) [6]:
where \( m \) is a model based measure defined over the same domain as \( p \). Now it is possible to incorporate spatial dependence between pixels in order to emphasize image’s features relevant for the understanding of the image by a human (contours, homogenous areas, etc.). A sensible choice is for example [5]:

\[
m_i = 1 + \sigma_i^2, \quad \sigma_i^2 = \frac{1}{N_3} \sum_{i \in N_3} \left( g_i - \mu_{N_3} \right)^2.
\] (6)

\( \sigma_i^2 \) is a grey-level variance over the 3x3 neighbourhood \( N_3 \) of the pixel \( i \) and \( \mu_{N_3} \) is the mean grey-level in \( N_3 \). The more variable an image, the greater the variance, so in this model the more random (or “noisy”) the image, the higher its entropy.

Entropic segmentation is based on an assumption, that the “entropy of each region is always lower than entropy of the whole image or, in other words, the entropy of a region is always greater than the entropy of its sub-domains” [7]. For the two classes of pixels (“black” \( b_0 \) and “white” \( b_1 \)) in an 8-bit greyscale source image we can calculate entropies as [2]:

\[
H_{b_0}(t) = -\sum_{i=0}^{255} \frac{p_i}{p(b_0)} \log \frac{p_i}{p(b_0)},
\] (7)

\[
H_{b_1}(t) = -\sum_{i=1}^{255} \frac{p_i}{p(b_1)} \log \frac{p_i}{p(b_1)},
\] (8)

where:

\[
p(b_0) = \sum_{i=0}^{1} p_i,
\] (9)

\[
p(b_1) = \sum_{i=1}^{255} p_i.
\]

The entropies \( H_{b_0} \) and \( H_{b_1} \) may be analogically calculated using eq. 5. by including spatial information. The segmentation criterion may be calculated in numerous ways, for example [8,9]:

\[
t = \arg \max \left( H_{b_0}(t) + H_{b_1}(t) \right)
\] (10)

or:

\[
t = \arg \min \left( H_{b_0}(t) - H_{b_1}(t) \right)^2.
\] (11)

4. Discussion

Entropic segmentation does not require any pre-processing or user interaction. It is able to identify “natural” classes present in an image. It is a
great advantage, because it is very difficult to formulate strict segmentation rules for medical images. However, this flexibility can also be a drawback.

Fig. 3 presents a CT head image and its histogram. Either entropic thresholding or any of histogram analysis methods could misclassify voxels with intensities close to the arrow mark (Fig. 3b) as one of head tissues. Actually, they belong to the pillow, patient’s head was placed on during the examination.

![Fig. 3. CT image and a fragment of its histogram](image)

Another problem is that various tissues happen to correspond to the same or similar voxel intensities. Fig. 4 presents a MR image and a thresholding segmentation result. It is observable that there are some tissues that generate signals (voxel intensities) similar to those of eyes (eyes segmentation is a common step in radiotherapy planning).

![Fig. 4. A MR image and a thresholding result](image)
The above examples show the need to introduce some expert knowledge into segmentation algorithms. This can be done in a number of ways. We propose a variation of an atlas-based method. Fig. 5 presents a MRI dataset and a robustly simplified image (an atlas template) of the object to be spatially localized (eyes). Then we perform an image registration procedure in order to find a fragment of the MR image, that is the most similar to the template. The registration framework constructed by the authors (described in detail in [10]) is based on maximization of mutual information [11]. One of the images (the template) is transformed with a rigid body transformation. During the optimization process the Powell’s algorithm [12] with sub-sampling and randomly selected multiple start points is employed. This algorithm works automatically and requires neither preprocessing nor human interaction. Fig. 6 presents the result. The accuracy is not critical, since we do need only a rough solution.
Having information on the estimated eyes spatial position (radically better than in Fig. 4), it is possible to segment them precisely. In order to be classified as a member of a given class, a voxel is required not only to have certain intensity, but also to be localized in an area where the given tissue is likely to occur and have at least a few neighbours of similar intensity (otherwise, it is likely to be an effect of noise or an artefact). It can be expressed by a fuzzy rule:

\[
\text{IF (voxel intensity is about } X) \quad \text{AND (voxel is close to the centre of one of the eyes in the template)} \quad \text{AND (voxel has a few neighbours of similar intensity)} \quad \text{THEN (voxel belongs to eyes)},
\]

where \( X \) is the average eyes voxels intensity. The fuzzy sets used in the above rule are presented in Fig. 7. These are a Gaussian curve, Z-curve and a sigmoid. The parameters have been empirically found. The operators are conventional ones, defined by Zadeh [13]:

\[
\begin{align*}
A \cap B &= \min(\mu_A[x], \mu_B[y]) \\
A \cup B &= \max(\mu_A[x], \mu_B[y]) \\
\sim A &= 1 - \mu_A[x].
\end{align*}
\]

Unclassified voxels are assigned an average membership value of their neighbours. The final result is presented in Fig. 8.

Fig. 7. The exemplary fuzzy sets used for segmentation: a) voxel intensity is close to \( X \), b) voxel is (spatially) close to one of the eyes in the atlas image, c) a few similar neighbours

Fig. 8. The segmentation result
5. Conclusions

It has been shown, that it is difficult to obtain satisfactory segmentation results by applying a single algorithm. On the other hand, a combination of various (often very distant) methods and expert knowledge may lead to much better solutions. The presented framework is still being developed. The part that has been already implemented is not only a theoretical issue. Eyes segmentation is often the first step in radiotherapy planning (eyes are very sensitive to X-radiation) and its automation is much desired. However, the aim is to achieve a full multi-class segmentation. The most problematic aspect is the construction of a representative atlas.

References