



Models of multichannel interconnection systems

Mirosław Hajder^{*}, Paweł Dymora, Marek Bolanowski

*Department of Distributed Systems, Faculty of Electrical and Computer Engineering,
Rzeszów University of Technology, Wincentego Pola 2, 35-959 Rzeszów, Poland*

Abstract

This work begins with presentation of modern teleinformatic systems functional problems. The authors indicate the necessity of using the multichannel communication technologies. Next, the building and exploitation costs analysis of interconnection systems was shown. Methodology of including designing system exploitation costs during the designing process was proposed. In the last part of the work the model allowing to design teleinformatic systems with expanded interconnection network more efficient was presented.

1. Introduction

One of the main development trends of teleinformatic systems observed in the past decade is integration of computer and telecommunication networks. Owing to this, in teleinformatic networks services typical of telecommunication networks, such as phone calls, teleconferences or video on demand become available. A variety of new, specialized communication protocols is used to implement those services.

Integration of two different types of communication networks aims at reducing cost of building and exploitation of company teleinformatic system. Building of a few dedicated networks (computer, telephone, television) is not necessary in this case, because the same devices and wiring may be used. Besides, administration of the whole system may take place from the same level of management program, by the same employee.

However, this solution has one essential shortcoming – it demands much larger system capacity than in the case of typical computer network. If for data transmission speeds below 1Gb/s are mostly sufficient, implementation of multimedia services in network may require increasing of its capacity to tens of Gb/s, and in some cases even to Tb/s.

^{*}Corresponding author: *e-mail address*: mhajder@prz.rzeszow.pl

Reaching such transmission speeds using technologies based on electrical media meets major limitation related with condition of transmission parameters from frequency of carrying signal. Using signal frequency greater than 1GHz causes major increase of attenuation of transmission media and distortion level. Eventually the maximum distance that signal can be sent does not exceed a few hundred meters.

Because of this, for building high-capacity, long-distance communication systems, mostly optical communication systems are used. Besides high capacity (theoretically even several Pb/s) these systems are immune to typical electromagnetic interference. Also fiber low attenuation allows to send signal without regeneration for distance of a few hundred kilometers.

The most popular kind of optical communication used in optical networks is multiwave optical communication. It assigns data streams to different optical wavelengths and allows to multiply transmission capacity of the fiber a few hundred times. Multichannel optical communication technology WDM was created for needs of such data transmission. The possibilities offered by WDM allow to design complex logical topologies based on simple structures of physical interconnections [1,2].

Besides, using all possible capacity, multichanneling ensures improvement of many other characteristics of transmission system. Based on available virtual channels, various logical topologies can be created, including symmetrical and regular topologies. Owing to this the following tasks are simplified in the communication system: routing, channel loads balancing, efficiency improvement in using computing elements, etc.

For building multichannel optical systems it is necessary to create algorithms generating logical topologies to assign physical topology regarding quantity of available transmission channels in one physical fiber, optical waves conversion, routing in optical topology, network load, traffic characteristics, elements placement in nodes, building and exploitation costs. Difficulty with realizing the designing process is mostly a result of exponential dependence of designing process computational complexity from the size of designing task.

2. Building and exploitation costs of interconnection systems

One of the teleinformatic system designer basic goals is to choose a goal function used during designing. Currently used algorithms in many cases do not have an imposed goal function. Mostly used goal functions during the designing process of multichannel communication systems are: minimize \bar{h} weighted average of jumps number, maximize f_{\max} of congestion level in network, minimize d average delay of information traffic. However, these parameters characterize the network from the technical point of view, understandable for the system designer. It is necessary from the investor's point of view to include

building and exploitation costs. In most of the known designing algorithms these costs are peripheral or even omitted. From the authors' experience it appears, that in the Polish conditions annual exploitation costs are usually from over a dozen to tens of percentage of building data communication system costs. In turn, the designing costs are between 0.3 and 15 percent of investment value. Such high designing costs are related to necessity of geodetic service of the project and compensation for owners of the grounds on which investment takes place. It is obvious that including building costs, exploitation costs will be also handsome. That is why it is advisable to include in designing algorithm minimizing exploitation costs, and in some cases also design costs. At the beginning, let us consider the building costs of teleinformatic system.

Let us assume, that designing network can be represented with the graph $G(V, E)$, where V – nodes set of a graph, the number of set elements, i.e. the number of network nodes is n ; E – edges set of a graph, its power i.e. the number of communication channels is m . Let us additionally assume, that building cost of the node equals K_W , and building cost of the communication channel is K_K . In this case, the building cost K of the network is represented by the equation:

$$K(n, m) = nK_W + mK_K. \quad (1)$$

Costs represented by equation (1) refers to building a one-level physical network. But users mostly use fragments of logical network. That is why costs due for each user should be defined based on optical paths.

If the designed network has a hierarchical structure from a physical point of view, it usually consists of high-capacity core, whose building costs are much higher those costs of access networks.

Let us assume, that the number of core nodes is n_R , and the number of access network nodes is n_D . The number of communication channels in core and access network are equal to m_R and m_D , respectively. By analogy, building costs of core and access network components are represented as K_{WR} and K_{WD} , respectively. Building costs of core communication channels and access network communication channels are K_{KR} and K_{KD} , respectively. Then the expression calculating network building costs turns into:

$$K(n_R, n_D, m_R, m_D) = n_R K_{WR} + n_D K_{WD} + m_R K_{KR} + m_D K_{KD}. \quad (2)$$

Let us assume, that core topology is a complete network. Then its building cost is presented by the expression

$$K_Z(n_R, m_R) = \frac{1}{2} n_R (2K_{WR} + (n_R - 1)K_{WD}). \quad (3)$$

Complete topology building costs are the highest of all network architectures used. These costs mostly consist of communication channels costs. That is why this topology should not be used if individual building costs of the transmission

channel are high. Complete network is preferred for building networks based on logical channels.

According to the prior assignment, a correct design from the investor's point of view should include designing, building and exploitation costs. Let us denote design costs, whose parameters are specified in the array X , as $K_p(X)$, and building costs as $K_b(X)$. Let us notify, that building costs include nodes building costs and communication channels costs. Let us assume, that connection (x,y) will be built. Then its building costs are given by $\chi'(K(x,y))$. Next, the node i ($i \in V$) building costs are given by $\chi''(i)$. Let us also assume, that communication channel building costs are expressed by the following equation:

$$\chi' = \sum_{(x,y \in V)} d_{x,y} P_{x,y} \xi, \quad (4)$$

where $d_{x,y}$ – coefficient of communication channel building costs, $P_{x,y}$ – communication channel capacity, ξ – coefficient including length of the connections.

However, using building costs as a goal function during the designing process is not reasonable. In many cases the use of more expensive solutions works in significant increase of investment exploitation time. Therefore, more accurate solution would be the use for estimating the investment by investment amortization amount. This amount can be calculated by means of the following equation:

$$\begin{aligned} K_A(X) &= E_n (K_p(X) + K_b(X)) \\ &= E_n \left(K_p(X) + \left(\sum_{(x,y \in V)} \chi'(K(x,y)) + \sum_{i=1}^n \chi''(i) \right) \right), \end{aligned} \quad (5)$$

where: E_n – investment amortization coefficient.

Full costs $K(X)$ estimated during the designing process, including expression (5), can be written as:

$$K(X) = K_A(X) + \mathfrak{I}(X), \quad (6)$$

where: $\mathfrak{I}(X)$ – system exploitation costs.

Most teleinformatic systems have lifetime from a few to over a dozen years. Using expression (6) for calculating investment costs, mostly first year exploitation costs are used. However, the system long exploitation costs can change every year. For example, with the system aging failure rate grows and exploitation costs are increased by repair costs. Because the described phenomenon refers almost to all systems for design purposes we may assume its constancy through the whole system exploitation time. This simplification does not affect process quality, but it allows to limit complexity of the designing process.

From the investor's point of view the designing process is based on creation of such a structure, for which [1]:

$$\text{opt } K(X) = \min K(X) \tag{7}$$

that is:

$$\text{opt}[K(X)] = \min[K_A(X) + \mathfrak{I}(X)] \tag{8}$$

In teleinformatic systems, including multichannel interconnection systems, channel building cost changes with modification of its capacity. Furthermore, in multichannel systems we can observe that capacity depends on the system wave channels number – their increase in number causes the increase of network building costs. Therefore, in order to calculate building costs dependent on communication channels capacity more precisely, we define the additional function FK of system building costs. It has a form constant in intervals and is expressed by means of the following equation:

$$FK(P) = \sum_{i=1}^m (U(P - P_i) - U(P - P_{i+1}))d_i(q), \tag{9}$$

where: m – number of invariability intervals, P_i – function limits of invariability intervals ($i = 1, 2, \dots, m + 1$), $U = \begin{cases} 0, & P - P_i \leq 0 \\ 1, & P - P_i > 0 \end{cases}$, $d_i(q)$ – value of costs function

on its invariable segment $[P_i, P_{i+1}]$, dependent on implemented connections length and communication channel capacity. In the same way using a constant in intervals function, we may estimate building network nodes costs.

Including the above assignation, expression (5) may be presented in a form including connection between investment cost and capacity of edges connecting particular nodes:

$$\begin{aligned} K_A(X) &= E_n(K_p(X) + K_b(X)) \\ &= E_n\left(K_p(X) + \left(\sum_{(x,y \in V)} \chi'(FK(x,y)) + \sum_{i=1}^n \chi_p''(i)\right)\right), \end{aligned} \tag{10}$$

where: $\chi_p''(i)$ – building cost of the node i , providing service of communication line of P capacity.

Let us consider the teleinformatic system, in which various types of communication channels and different node architectures were used. Let us assume, that a network nodes set is represented by a set $V = \{1, 2, \dots, n\}$. Next, the network nodes set, in which user's resources are placed, is represented by a set $W = \{1, 2, \dots, \eta\}$. Let the connection between the nodes x and y ($x, y \in W, x \neq y$) be denoted as xy . Communication channels used in the designing system are characterized by step (discreet) changing of capacity. Let

us assume that κ channel types which differ in capacity, are used in the system. Then the discreet set R^K of channel types may be written as $\{R_1^K, R_2^K, \dots, R_\kappa^K\}$. In the considered system computing powers will be placed in the chosen nodes. There will be available l types of processing elements, differing in computing power. To represent computing powers, the units presented in [3] may be used. In this work the number of sets of basic process operations performed by the computing element in time unit is used. Let the number of computing elements types, characterized by different capacity, is l . Then the discreet set R^{CPU} of communication channel types may be presented by $\{R_1^{CPU}, R_2^{CPU}, \dots, R_l^{CPU}\}$.

Designing optical logic networks, one assumes that the optical path realization costs are proportional to the path distance, i.e. the number of hops between the source and the target node. This way of distance measurement is ineffective for physical networks, it is necessary to include real distances between nodes. Therefore, let us assume that the actual distance between the nodes x and y ($x, y \in W, x \neq y$) is L_{xy} . Then real distances between network nodes may be presented with the two-dimensional distance matrix $L^W = \|L_{xy}\|$, where $x, y \in W$, and if $x = y$ then $L_{xy} = 0$. Costs $K_{xy}^{K_i}$ of building communication channel of type i , connecting nodes x and y are the function of channel capacity R_i^K and distance L_{xy} between nodes. It is allowed to write, that $K_{xy}^{K_i} = f(R_i^K, L_{xy})$.

The second main component of communication system costs are nodes building and exploitation costs. Let us consider the node building and the exploitation costs K_i^{CPU} equipped with devices of type i . In this case, costs are dependent only on capacity R_i^{CPU} of the processing device used. Therefore, one can write $K_i^{CPU} = f(R_i^{CPU})$.

3. Models of teleinformatic system topologies

Designing the teleinformatic system topology is widely described in literature [4-7]. Most of the designing algorithms is based on the analysis of queuing systems of type M/M/1 and resolve to solving of the model described below.

Let us begin building the teleinformatic network with the connection structure described. Therefore let us introduce binary variables describing connections in the system. Basic characteristics of any teleinformatic system is communication channels structure. Mostly it is presented with the interconnection matrix $\Omega = \|\omega_{xy}\|$ describing communication channel existence between the nodes x and y . The nodes of the network being analyzed are assigned to columns and

rows of the matrix. Value of element ω_{xy} of matrix Ω is defined by the expression:

$$\omega_{xy} = \begin{cases} 1 & \text{if the node } x \text{ is connected with the node } y \text{ } (x, y = 1, \dots, n, x \neq y) \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

Because the teleinformatic system may use various types of communication channels, the variable ξ_{xyk} is introduced, which species the connection channel type connecting nodes x and y . This variable is described by the expression:

$$\xi_{xyk} = \begin{cases} 1 & \text{if the node } x \text{ is connected with the node } y \text{ using the channel} \\ & \text{of type } k \text{ } (x, y = 1, \dots, n, x \neq y, k = 1, \dots, K_k^k) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Matrix representation of variable ξ_{xyk} may be realized in a few ways. In the simplest case the three-dimensional matrix $\Xi(x, y, \xi_{xyk})$ can be used, in which the values ξ_{xyk} would be described using expression (12). Because matrix Ξ is a sparse matrix, memory resources would be used inefficiently in this method. An interesting solution would be connection of matrixes Ω and Ξ . Then the elements of modified matrix Ω would be described using the expression:

$$\omega_{xy} = \begin{cases} k & \text{if the node } x \text{ is connected with the node } y \\ & (x, y = 1, \dots, n, x \neq y) \text{ using the channel of type } k \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

This method guarantees computer memory efficiency usage, however the variable ω_{xy} loses here its binary character. Other methods of representing the channel type used may be based on matrix W and will not be described here.

In the modelled system it is necessary to link nodes with resources available in network. Matrix $\Delta = \|\delta_{xk}\|$ is used for this purpose, whose element δ_{xk} describes the location of the processor of type k in the network node x . Value of this binary variable is defined with the expression:

$$\delta_{xk} = \begin{cases} 1 & \text{if in the node } x \text{ the processor of type } k \text{ is located} \\ & (x = 1, \dots, n, k = 1, \dots, R_l^{CPU}) \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

One of the most important characteristics of teleinformatic system is routing, defining data transfer routes. Routing can be described by linking the data communication stream route with the individual system nodes, and then by choosing the shortest route connecting two indicated system nodes. For this

purpose let us introduce the matrix $Z_{xy} = \|z_{xy}(sd)\|$, whose elements are defined with the expression:

$$z_{xy}(sd) = \begin{cases} 1 & \text{if the packet from the node } s \text{ to the node } d \text{ is transmitted} \\ & \text{via channel including the node } x \text{ and the node } y \\ & (x, y, s, d = 1, \dots, n, x \neq y, s \neq d) \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

Because sizes of the modelled teleinformatic network may be considerable, it is necessary to elaborate an efficient structure of data used in the model. This structure must be readable and clear for the designer, and also optimal from the memory and the computing resources usage point of view. Data compression in one structure being the matrix Ω was suggested. It is assumed that the multistate matrix Ω variable is recorded on 8 bits. Let us assume, that the first seven bits describe the channel K type, used for connecting the nodes x and y , and 8th bit would be the matrix $\Delta = \|\delta_{xk}\|$ binary element. The new matrix $\Phi = \|\phi_{xy}\|$ may be defined by the expression:

$$\phi_{xy} = \begin{cases} k_7 k_6 k_5 k_4 k_3 k_2 k_1 \delta_{xk} & \text{if the node } x \text{ is connected with the node } y \\ & (x, y = 1, \dots, n, x \neq y) \text{ using the channel of type } k \\ 0_7 0_6 0_5 0_4 0_3 0_2 0_1 \delta_{xk} & \text{otherwise} \end{cases} \quad (16)$$

To use the field of δ_{xk} variable more efficient, the column numbers in the matrix $\Phi = \|\phi_{xy}\|$ are assumed to represent type of processor. So when the variable δ_{xk} has the value equal one in the row denoted with number 2 in the column denoted with number 3, one can conclude that the processor of type 3 is placed in node 2.

Figure 1a shows the matrix $\Phi = \|\phi_{xy}\|$ logical schema, Figure 1b shows its construction, and figure 2 shows an example of topology described with the matrix $\Phi = \|\phi_{xy}\|$.

One of the most important problems solved during designing a network stage is to state routing policy. Let us assume, that in our network it is achieved based on the shortest path algorithms. For this purpose let us introduce the matrix of shortest paths connecting the nodes x and y . Let us denote this matrix as $\Psi_{xy} = \|r_{xy}\|$, and matrix elements would be described by variables $z_{xy}(sd)$ and ω_{xy} . Let us notice, that the number of paths connecting two nodes can be considerable. Therefore it is necessary to limit the number of considered paths.

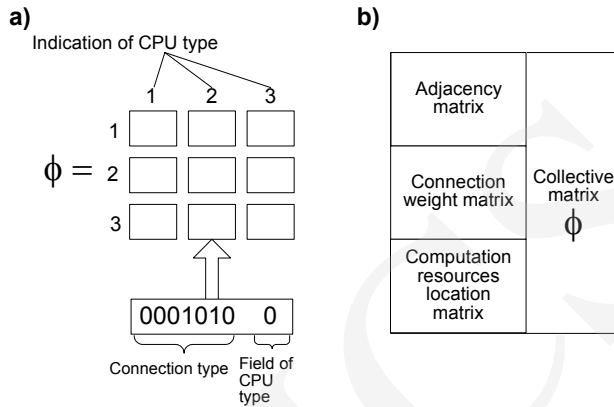


Fig. 1. a) Matrix $\Phi = \|\phi_{xy}\|$ logical schema, b) matrix $\Phi = \|\phi_{xy}\|$ construction

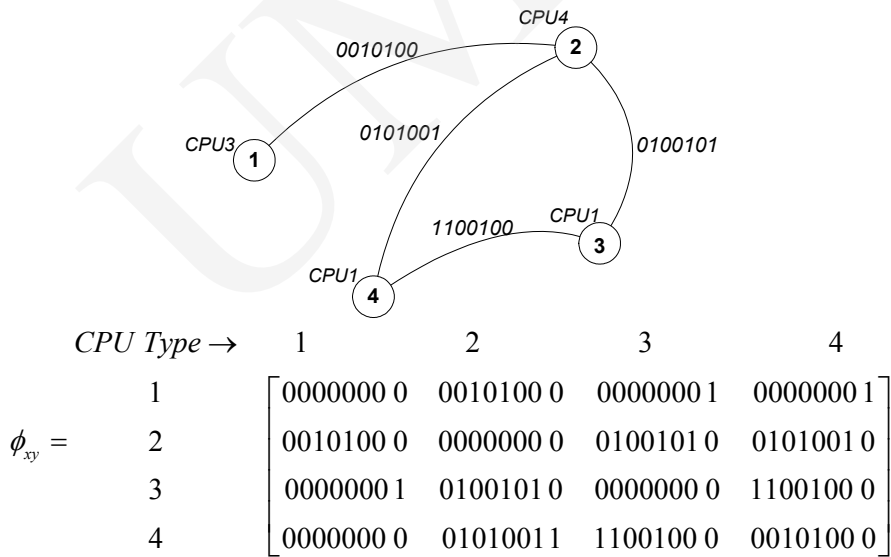


Fig.2. Example of a topology and its matrix $\Phi = \|\phi_{xy}\|$

Let us assume, that this limitation is equal to P_{xy}^K . Choosing the limitation value should be made aware of because network architecture and parameters are dependent on it. On the other hand, analyzing too many alternative paths leads to major increase of designing process computing complexity.

From the communication point of view, the analyzed teleinformatic system is characterized by such parameters: the information streams intensity as well as

their average length. The information stream intensity between the nodes x and y ($x, y \in W, x \neq y$) is denoted as γ_{xy} . It characterizes the frequency of information streams appearance in a given communication channel. This parameter, characteristic of each of existing channels in the whole network, describes the matrix of streams intensity $\Gamma = \|\gamma_{xy}\|$. Next, defined for each channel, the average length of information streams is the average amount of bytes transferred in the confines of one information stream. This parameter for the channel connecting the nodes x and y is denoted as $\bar{\mu}_{xy}$.

From the processing point of view the analyzed system is characterized mainly by the computation complexity Z_i of one information stream byte processing in the node of type I and expressed in bytes of the average length of information stream \bar{D}_x processed in the node x .

In order to improve the system functional parameters, its nodes are equipped with buffer memory whose main purpose is buffering of information streams in the node. The capacity of buffer memory located in the node x is denoted as C_x . The last network parameter is the maximal allowed value of average network delay denoted as \bar{T}_{\max} . All of the above parameters describe the requirements placed by network architecture. Now we can focus on creating a system model.

At the beginning, we are interested in the summary intensity of information streams transferred in the network. In order to do this, the above presented variable γ_{xy} which describes the information stream intensity between the nodes x and y will be used. The intensity of information stream Γ_x addressed to the node x is described by the expression:

$$\Gamma_x = \sum_{y=1}^n \gamma_{xy} \quad x, y = 1, \dots, n, x \neq y. \quad (17)$$

Next, the intensity of information stream B_{xy} between the nodes x and y is described by the expression:

$$B_{xy} = \sum_{s=1}^n \sum_{d=1}^n \gamma_{sd} \omega_{xy} z_{xy}(sd) \quad x, y = 1, \dots, n, x \neq y, z_{xy}(sd) \in R_{sd}. \quad (18)$$

The intensity of information stream reaching the node x :

$$\Phi_y = \sum_{x=1}^n B_{xy} \omega_{xy} \quad y = 1, \dots, n, x \neq y. \quad (19)$$

In the end the summary intensity of information stream is described by the expression:

$$S = \sum_{x=1}^n \Gamma_x. \quad (20)$$

The mathematical model of optimization task is based on the values of structural parameters which can be expressed with the vector of structural variables $\sigma = (\omega_{xy}, \xi_{xyk}, \delta_{yk}, z_{xy}(sd))$. Then the cost function K can be expressed:

$$K = \min_{\sigma} \left[\sum_{x=1}^n \sum_{y=1}^n \sum_{k=1}^{\kappa} K_{xy}^{K_k} (R_{xy}^{K_k}, L_{xy}) \right] \quad (21)$$

$$\frac{\mu_{xy} B_{xy}}{R_{xy}^{K_k}} \left[\omega_{xy} \xi_{xyk} + \sum_{y=1}^n \sum_{k=1}^l K_{yk}^{CPU} (R_{yk}^{CPU}) \right] \frac{\Phi_y Z_k \bar{D}_y}{R_{yk}^{CPU}} (\delta_{yk}).$$

The system parameters are estimated under the assumption that the communication channels and processing elements are described with the massive service model of type M/M/1. Using this model the following restrictions must be obeyed:

The network should be coherent, and the number of routes between each pair of nodes x and y should not be smaller than that specified for a given pair of values P_{xy}^K :

$$|\Psi_{xy}| \geq P_{xy}^K, \quad x, y = 1, \dots, n, \quad x \neq y \quad (22)$$

Memory size used for buffering messages at the node j should not be greater than the size specified for a given node:

$$M_y \bar{D}_y \leq C_y, \quad y = 1, \dots, n \quad (23)$$

where: M_y – number of packets waiting for a service at the node y specified for a massive service model M/M/1.

In order to make a connection between the network nodes x and y , it is necessary to use the same type of channels:

$$\sum_{k=1}^{\kappa} \xi_{xyk} = 1 \quad x = 1, \dots, n, \quad x \neq y. \quad (24)$$

Capacity of interconnection channels and the computation performance of processing elements of a given node should outnumber the intensity of transmitted and processed information stream, respectively:

$$\sum_{k=1}^{\kappa} \frac{R_{xy}^{K_k} \xi_{xyk}}{\mu_{xy}} - B_{xy} > 0 \quad x, y = 1, \dots, n, \quad x \neq y \quad (25)$$

$$\sum_{k=1}^l \frac{R_{yk}^{CPU}}{D_y Z_y} \delta_{yk} - \Phi_y > 0 \quad y = 1, \dots, n$$

The average delay \bar{T} of packets at network should not exceed the earlier specified value \bar{T}_{\max} , where:

$$\bar{T} = \frac{1}{S} \left(\sum_{x=1}^n \sum_{y=1}^n \sum_{k=1}^K \frac{B_{xy} \mu_{xy}}{R_{xy}^{K_k} - B_{xy}} + \sum_{y=1}^n \sum_{k=1}^l \frac{\Phi_y \bar{D}_y Z_y - (\Phi_y)^2 \bar{D}_y}{R_{yk}^{CPU}} \right). \quad (26)$$

The presented model describes characteristics of optical communication the system of bus communication. However, there is no objection to description of other topologies (bus, star, etc.) using this model.

4. Summary

The process of convergence of teleinformatic services caused, that at present it is necessary to build high-efficient, reliable networks, which can meet demands. It is necessary to build full optic computer networks using WDM technology. Designing process complexity causes that it reaches far beyond designer's intuition abilities. For such complex solutions, network designing algorithms should be developed, excluding designer's intuition to a minimum and including as many input parameters as possible. Developing a mathematical network model also allows to test the obtained solution before its implementation. Treatment of designing WDM systems presented in this work is one of the first efforts of designing network with simultaneous including efficiency parameters of the network and its construction as well as exploitation costs. The authors present the original methods of determining the computer network construction and exploitation costs, and implement them in the presented model of computer interconnections topology.

References

- [1] Hajder M., Bolanowski M., Paszkiewicz A., *Designing of improvement efficiency of multi-channel optical communication system criteria optimization*, Annales Informatica UMCS, 1 (2003) 271.
- [2] Hajder M., Dymora P., Mazurek M. *Improvement methods of multichannel bus topologies parameters*, Wydawnictwo Wyższej Szkoły Informatyki i Zarządzania w Rzeszowie, (2002).
- [3] Hajder M., Loutskaa H., Stręciwilk W., *Informatyka – Wirtualna podróż w świat systemów i sieci komputerowych*. Wydawnictwo Wyższej Szkoły Informatyki i Zarządzania w Rzeszowie, (2002), in Polish.
- [4] Elias D., Ferguson M.J., *Topological Design of Multipoint Teleprocessing Networks*, IEEE Transactions on Communications, 22(11) (1974) 1753.
- [5] Gerla M., Kleinrock L., *On the Topological Design of Distributed Computer Networks*, IEEE Transactions on Communications, 25(1) (1977) 48.
- [6] Boorstyn R., Frank H., *Large-Scale Network Topological Optimization*, IEEE Transactions on Communications, 25(1) (1977) 29.
- [7] Ersoy C., Panwar S.S., *Topological Design of Interconnected LAN/MAN Networks*, IEEE Journal on Selected Areas in Communications, 11(8) (1993) 1172.