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Use of simulation techniques for profit forecasting of saving programs

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Abstract

The article shows simulation techniques as a tool for extrapolation of long-term financial variables to support choice of savings form. Special attention has been paid to techniques used in the case of incomplete information or lack of it, that is in uncertainty conditions, accompanying modelling of system behaviour in the distant future. The experiment has been performed on the stochastic model in which both some stimulants included and also decomposed in time course of reaction are of random charcter.

The simulated dependent variable is the future level of savings which is possible to obtain after investing money for many years in a chosen form of secure capital investment. Whereas the set of stimulants includes number of financial variables and those describing standards of saver living. In this article the author focuses on interpretation of the obtained simulation results.

Theoretical reflections and conclusions that arise from the analysis of performed simulation are included in listing of positive and negative effects of using simulation techniques in the process of of investment decisions rationalization.

1. Introduction

One of the most important circumstances accompanying decision making is evaluation of its effects. Effect evaluation can be done in two ways. The first one is planning and performing using real system and physical experiment consisting in implementing a chosen variant of the decision in question and monitoring system reactions. However, this way is expensive, work and time consuming. Furthermore, if effects of the experiment are unfavourable, it can be also dangerous for the environment, in which it has been carried out. The second possibility to evaluate effects of the choice made is to perform simulation consisting in performing an experiment using a specially built model, that copies chosen aspects of the real system.

This article is aimed at presenting simulation methods as tools, that make it possible to create a scenario of financial variables development in a long period

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of time and to gather information that can support choice of savings form. In this article the author focuses on simulation results interpretation. The model, that has been built, is based on safe investments forms with special consideration of Individual Retirement Account IRA[1].

2. Selection of simulation method

The simulation model consists of dependent variables that respond to the decision and independent variables that function as tools decision accomplishment.

Assuming that the aim of the research is the variable y included in the system, the model of the system in relation to y can be generally written as:

$$y = \Psi(x_1, x_2, ..., x_k) \tag{1}$$

where: y reaction (dependent variable), x_k *for* $k = 1,2,...,K$ stimulants (independent variables), Ψ reaction surface (kind of dependence of variable y on variables x_k).

Complexity of the model depends on complication degree of the problem, that will be simulated. In the analyzed example the dependent variable is profit level, whereas in the set of stimulants where there were taken: disposable income of the saving person[2], saving tendency of the decision-maker, average forecasted salary in the country per month, real investment rate of return. The response surface is formed by: relations arisen from the financial maths theory, additional assumptions determining connections between variables related to the saving person standard of living and conditions concerning dynamics of changes. Specificity of long-term financial values forecasts causes, for uncertainty sources in some fragments of the created model. They have to be described by introducing random values and disturbances of variables, parameters or restriction courses. The fact, random stimulants and/or restrictions appear in the model makes it the stochastic model unlike the deterministic one, which does not include disturbances.

Simulation with the use of Monte Carlo (MC) method, which is described[3] as the "choice technique of random quantity from certain probability distribution", is an example of the stochastic simulation. There are several fundamental reasons determining the Monte Carlo methods to be an important pillar, on which computer simulations of the system behaviour are based. Among others, there are[4]: possibility of making observations of proceeding evolution of the examined occurrence or variable shaping; ease of MC methods use considering both mathematical and technical aspects (easy numerical implementation);

[1]In polish: IKE (Indywidualne Konta Emerytalne)
[2]Most important objective indicator of material prosperity is disposable income - Por. [1]
[3]Por. [2], s. 4
[4]Por. [3], s.21

possibility to obtain in a short period of time multiplied amount of information that can be used to build forecast for development of occurrence in question; finding mechanisms ruling the occurrences in question.

Particularly essential and problematic in the long-term economical simulations is necessity for determination of probability distribution for every random variable. Estimation of parameters for such distribution based on the examined system (variable) of behaviour observation is usually difficult or even impossible because of lack of empirical material satysfying forecast assumptions for a long period of time. In this case, kind of distribution and its parameters are most often based on experts opinion and/or based on assumptions. In this experiment this assuption was accomplished with the use of triangular distribution, normal distribution and additional variable restrictions.

3. Simulation model construction[5]

For a potential client choosing long-term money investment knowledge of anticipated profit and final savings account balance is essential. To estimate this from a long period of saving, series of initial assumptions and the use of simulation techniques are required. For this examination a model was built and it was based on three options of cyclical saving:

Option 1. Client deposits money on Individual Retirement Account (IRA). In the case, when as a result of yearly limitation he can not deposit in the IRA the whole sum he planned to save, then he deposits only as much as limit allows and resigns from investing the rest (surplus) using it for current consumption (additional surplus is not invested in any other form).

Option 2. Client uses two saving forms: deposits money on the Individual Retirement Account and he deposits surplus, that is left because of yearly limit of payment on the IRA, to another rated[6] savings program.

Option 2. Client uses another than the IRA savings form, which does not have any payment limitations. Therefore he can deposit to the chosen savings program the whole sum that he planned to save.

In order to compare profit[7] level, that can be obtained from depositing money according to the principles from Option 1, Option 2 or Option 3, there was performed a simulation based on the following assumptions:

1) monthly program payment is dependent on salary of a person taking part in it and is not higher than 10% of the person's disposal money,
2) salary (amount of money that is monthly at the disposal) of a person that wants to make savings, changes according to normal distribution

[5]The model was partly taken from por. [1]

[6]According to law, tax rate on capital profit is 19%.

[7]In the article profit will be presented as difference between final account balance and a sum of all payment.

(parameters $\mu = 0.05$ and $\delta = 0.02$) of the random variable X_1 which represents monthly rate of salary increase,
3) every 8 months the decision-maker has a chance to get a rise with probability of getting it equal to 0.3,
4) average salary in the national economy that is announced every year rises according to the triangular distribution (parameters $\alpha_2 = 0.01$, $\beta_2 = 0.03$, $\gamma_2 = 0.04$) of the random variable X_2 which represents the yearly rate of average salary increase in Poland;
5) real yearly rate of return from investment is the random variable X_3 with the triangular distribution (parameters $\alpha_3 = 0.03$, $\beta_3 = 0.05$, $\gamma_3 = 0.07$),
6) financial means are multiplied according to composite capitalization assumptions with the defined monthly effective rate of interest and payment in advance.

In the first and second Options it was also assumed that the decision-maker is going to use the whole IRA yearly payment limit unless his financial situation allows it. Furthermore, it was assumed that the withdrawal form savings program will be made when the person is 60 years old. It can be assumed, that simulation includes the person, who starts the savings process in the third pillar with intending to use the accumulated money for pension.

Simulation, described by the assumptions mentioned above, was performed for $M = 18$ variants of starting salary (disposable income at the moment when the person starts savings), in turn for $K = 3$ Options. In this way, it was obtained $M \times K$ ($3 \times 8 = 54$) final values of profit whose level is dependent on the option choice and initial income. The salary in consecutive variants readmit value from 1600 PLN ($m = 1$) to 5000 PLN ($m = 18$) with a step at every 200 PLN. The simulation process was repeated $n = 1000$ times, which gave 1000 consecutive replications of the examined data. The following notations were used in simulation:

– *k.m* – combination compounded from k- Option and m- variant of starting salary ($m = 1,2,\ldots, M$; $k = 1,2,\ldots, K$),
– $Z_{k.m}$ – profit level for k- Option and m- variant of starting salary ($m = 1,2,\ldots, M$; $k = 1,2,\ldots, K$).

From the results analysis point of view, it is essential that the simulation was performed with the same assumptions I-IV for all options and variants. This situation allowed for effectiveness comparison between the examined options and showed differences in profit level for particular combinations “k- option – m- variant”. Presentation of profit as value instead of in percentage terms as a rate of return is legitimate if it is assumed, that the sum of all payments is constant regardless of the savings form. In the simulation in question in the case of equal initial income (for particular *m*) this condition is met. Whereas for different values of initial income, the comparison of absolute profit levels is of research purpose only if it is performed in range of one option (for particular *k*).

It allows to measure profit sensitivity (expressed in PLN) to a change of income at the moment when savings starts for particular option.

4. Profit value in relation to disposable income – simulation results analysis

Financial situation of the saving person determines considerably the level of profit possible to achieve. Profit, that can be achieved after 20-year saving in the chosen program according to the initial income variant, was analyzed in order to stress the differences in final results according to the initial income. The obtained results for three chosen variants in the range of consecutive options are presenting in Table 1.

Table 1. Profit distributions $Z_{k.m}$ for Options k = 1,2,3 and initial salary variants 1800 PLN (m = 2), 2800 PLN (m = 7), 3800 PLN (m = 12).

	$Z_{k.m}$	Distribution : (μ,σ)	$E(Z_{k.m})$	$D^2(Z_{k.m})$
Option 1	$Z_{1.2}$	LN : (10.47;0.087)	35512.18	9643671
	$Z_{1.7}$	N : (54759.25;4241.98)	54759.25	17994404
	$Z_{1.12}$	N : (60629.18;3263.26)	60629.18	10648886
Option 2	$Z_{2.2}$	LN : (10.471; 0.087)	35512.18	9643671
	$Z_{2.7}$	LN : (10.914; 0.085)	55132.82	21991528
	$Z_{2.12}$	LN : (11.175; 0.078)	71518.51	30931071
Option 3	$Z_{3.2}$	LN : (10.197; 0.0863)	26932.79	5418968
	$Z_{3.7}$	LN : (10.639; 0.0863)	41895.45	13112564
	$Z_{3.12}$	LN : (10.945; 0.0863)	56858.11	24151204

N – normal distribution; LN – log-normal distribution.
Distribution compatibility has been verified with the use of Chi-square test.

The person deciding that part of the income will be deposited on Individual Retirement Account (Option 1), has to be aware of the fact, that yearly level of payment is statutorily limited and equals to 1.5-multiplicity of forecasted average monthly salary in the national economy, which is announced in a given year. There is a risk that persons who have greater amount of disposable financial means will not be able to deposit the whole sum on the IRA. This fact is shown by the relations between the anticipated profit values $Z_{1.2}$, $Z_{1.7}$ and $Z_{1.12}$ (Table 1). The anticipated value $E(Z_{1.7})$ is by 54% higher than the anticipated value $E(Z_{1.2})$. However, a saving person, whose initial income is 3800 PLN, can expect profit $E(Z_{1.12})$ larger than $E(Z_{1.7})$ only by 11%. Since the higher is the decision-maker income, the higher payments will be made on the account. It causes that a person in better financial situation can use the yearly limit of payments already in the first months of investment. The amount of money that in this case the person is prone to deposit on IRA next months is its financial surplus. For combination 1.2 probability of having surplus in the whole period of saving is equal to zero (maximal value of surplus sum is for the set of thousand repetitions equal to zero), and for combination 1.12 equal to zero is the probability of not having surplus (minimal value of surplus sum is for the set of thousand repetitions positive). Taking upper (optimistic) limits used in variables simulation, the total sum of surplus in the variant m = 12 (3800 PLN) in the whole period can achieve the value even close to 68000 PLN. Therefore together with the initial income increase, the frequency of surplus occurring and its value increase, while the profit growth rate from the IRA decreases.

Effects of payment restrictions in Option 1 reveal in the process of matching distributions to the resultant values $Z_{k.m}$. Distributions describing the profit trajectory $Z_{k.m}$ have been compatible with the log-normal distribution with an exception of the two combinations: 1.7 and 1.12. Distribution symmetry of the variables $Z_{1.7}$ and $Z_{1.12}$ is caused by the fact, that probability of achieving higher profit values is in option 1 limited by the IRA payment restrictions. Option 1 does not anticipate additional surplus deposits in another saving form. Therefore there are no characteristics for right-sided asymmetrical profit distributions, so called "right tails", converging high variable values. Whereas in combination 1.2 financial surpluses were not recorded and the whole amount of money assigned for savings could be deposited on the IRA. The random variable distribution $Z_{1.2}$ has typical of profit right-sided asymmetry, which suggests that there is positive although small probability of occurring of extremely high profit values.

The average amount of money, which due to restrictions, could not be deposited on the IRA, increases with every increase of initial income. However, this increase occurs in the analyzed example only for the income value above 2800 PLN (m = 7). At the lower initial income values, surpluses were noted on the level of central tendency positional measures which divided collectivity into smaller parts than median.

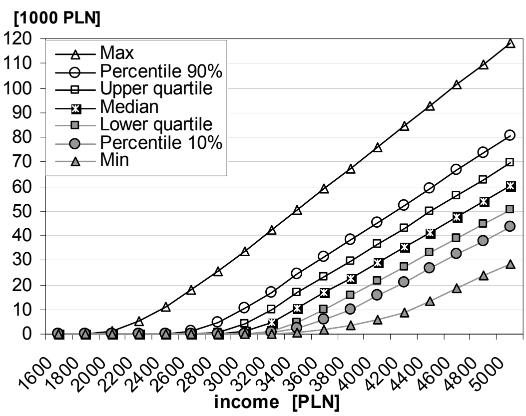

Fig. 1. Surpluses values [in thousand PLN] dependant on initial income for the 20-years period of saving

Observing development of the profit level $Z_{1.m}$ for consecutive income variants (Fig. 2a), such an initial income level, after which profit growth rate considerably decreases can be seen. It follows from the graph, that it happens for the income value about 3000 PLN. Occurrences of increased surplus values together with the income increase and decrease of profit rise indicate, that between the client income and the IRA profit there exists certain relation: if the monthly IRA payment amount is expressed as the constant percent of the saving person's income, then assuming the constants of remaining factors there exists such an income limit, that with every further income increase, profit level rise is becoming smaller and smaller. Moreover, if the yearly IRA payment limit is completely used by the first monthly payment, then further income rise will not bring the IRA profit increase. Therefore the problem of financial means saving, which due to statutory restrictions can not be deposited on the IRA, appears. In this case the decision-maker can choose other safe capital accumulation forms.

a) Option 1 ($Z_{1.m}$) b) Option 2 ($Z_{2.m}$)

c) Option 3 ($Z_{3.m}$)

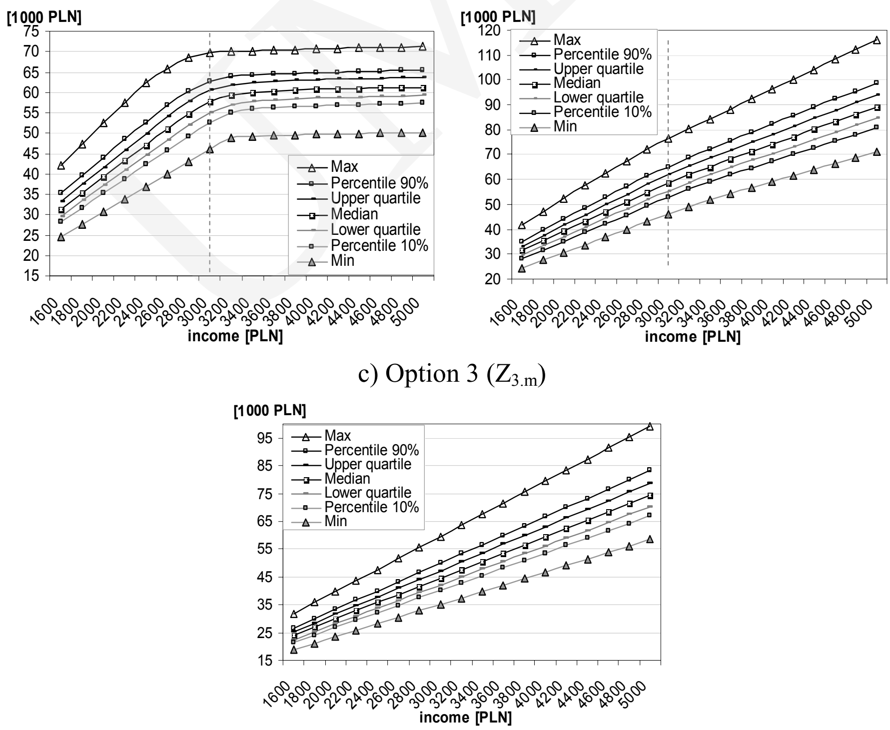

Fig. 2. Profit level after 20-year saving in relation to the initial income variant *m* and the Option *k* (k = 1,2,3; m = 1,2,…,18)

Assuming, that savings will be multiplied according to Option 2, that every surplus will be deposited on other than IRA saving program, profit which the saving person will make after 20-year saving will be partly derived from the IRA and partly from another program. Such a solution is of significant importance for the people, whose incomes are high, because every sum, that can not be deposited on IRA will, anyway, generate profits owing to its deposition in another form. Occurrence of surplus is already possible (although with small probability) by the initial income at 2000 PLN ($m = 3$). Surpluses sum is however small and does not influence significantly on future profit level.

Comparing the anticipated IRA profit values ($Z_{1.m}$) with the total profit received from the IRA program and from the additional surplus depositing ($Z_{2..m}$) for the three chosen initial income variants $m = 2$, $m = 7$, $m = 12$ there were obtained the following relations[8]:

$$E(Z_{2.2}) = E(Z_{1.2}), \tag{2}$$

$$E(Z_{2.7}) > E(Z_{1.7}), \tag{3}$$

$$E(Z_{2.12}) > E(Z_{1.12}). \tag{4}$$

The sign of equality is between the anticipated profit values $Z_{2.2}$ and $Z_{1.2}$ because at the initial income on the level of 1800 PLN there are no financial surpluses. Part of income, that the person wants to save, can be in this case as the whole deposited on the IRA because imposed yearly payment limit will not be exceeded. The profit distribution $Z_{2.2}$ is identical to the log-normal distribution of profit $Z_{1.2}$. The signs of inequality in relations (3) and (4) result from surpluses occurrence for the income variant $m = 7$ (2800 PLN) and $m = 12$ (3800 PLN).

Analyzing the profit levels in Option 2 for m = 1,2,…,18 (Fig. 2.b) there can be seen – similar to Option 1 – such an initial income level, after which the profit growth rate decreases. However, there exists in Option 2 such an initial income level, after which profit growth rate stops diminishing and stabilizes on the particular level. Therefore in Option 2 there should be pointed to the range of initial income, in which by assumption of constant remaining factors with every consecutive salary increase, the profit level increases is less and less. Comparing to Option 1, the profit growth rate decrease is not very significant. Diminishing profit increase along with income increase is the result of reducing IRA profit proportion to that from the taxed saving form. The larger part of income has to be deposited in the taxed program, the higher alternative cost of lost interests. However, if the yearly IRA payment limit will be maximally used by the first monthly payment, further income increase causes constant total profit increase driven only by the interest increase from the taxed program. In Option 2, because of lack as in Option 1 of yearly payment limit (any surplus that occurs can be deposited in other than the IRA program), every income increase results in profit increase.

[8]Look Table 1

Comparing the shape of lines that illustrate the profit level changes $Z_{2.m}$ and $Z_{1.m}$ there can be seen a distinct difference in the curves slope for the variants, in which after depositing money in the IRA some free financial means remain. For this variants inclination angle to the x-axis of tangent to the average profit curve in any point is higher than in Option 2. In Option 1 due to higher initial income values this angle diminishes to zero because there is average profit curve limitation by the horizontal asymptote.

Therefore both in Option 1 and Option 2 the profit level growth rate is dependent on the initial income and changes in the examined range. Constant and independent of the initial income, the growth rate occurs in Option 3, in which it is assumed that a saving person will not use the IRA form at all but all money will be deposited in the program that has taxed interest rates. In Option 3 there are no payment restrictions, that could slow down the profit growth rate together with the income increase and cause, that the line of average profit takes the logarithmic shape. The graphic presentation of average profit level in Option 3 for the particular initial income variant are the points, through which there crosses a straihght line straight line with the directional equation:

$$y = 0.014884x, \quad x \in R .$$

Based on the directional factor it can be said that increase of 200 PLN of the initial income will result in the average profit increase by 2976.9 PLN.

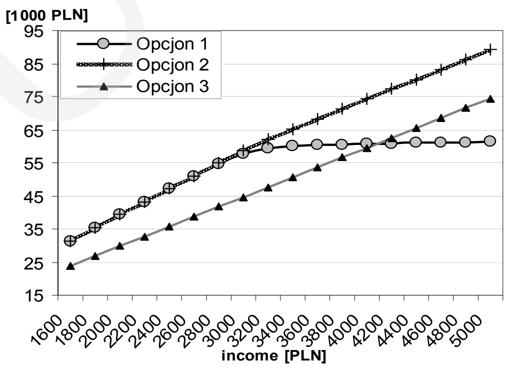

Fig. 5. Average profit value $Z_{k.m}$ achieved after 20-year saving for k = 1,2,3; m = 1,2,…, 18

The third option does not look favourably compared to the alternative solutions. Necessity of tax payment from the capital profit causes, that for the chosen initial income variants m = 2, m = 7, m = 12, the anticipated profit values in Option 3 are not only lower than the anticipated values in Option 2 but also than in Option 1[9]:

$$\mathrm{E}\left(Z_{3.2}\right) < \mathrm{E}\left(Z_{1.2}\right), \tag{5}$$

[9]Look Table 1

$$E(Z_{3.7}) < E(Z_{1.7}), \tag{6}$$

$$E(Z_{3.12}) < E(Z_{1.12}). \tag{7}$$

The average achievable profit after 20-saving in Option 3 is higher than the average IRA profit only for the income variant $m = 14$ (initial income equals 4200 PLN) and is lower than the average profit in Option 2 regardless of the initial income (Fig. 3).

Conclusions

In the decision making process related to the long term saving forms, the use of simulation techniques is indispensable. The operating simulation model gives possibility to gather information about states, which due to the assumptions can become reality in the future. However unique economic variable character, which is essential for result precision, leads to the conclusion, that using simulation methods is also loaded with certain risk. The more precisely the real system is imitated in the model, the lower is the risk.

In the case of long term economic variable experimental forecasting, the extrapolation mechanism correctness in the model can be verified only *ex post.* In the construction phase there is lack of reliable and verified information concerning future. Definition of uncertainty borders, for variables development in time, is in this case a superior task.

With lack of suitable empirical material, constructing model is based on economic premises and experts opinions. This fact from the beginning indicates existence of uncertainty in assumptions. Still, including random disturbances in the model should be done to a moderated extent. Too many of them can cause, that the outgoing data information value will be low because of disproportionate relation between the information increase and its effectiveness[10].

The experiment presented in the article, which used simulation techniques for profit forecasting, was performed using the stochastic model that considers uncertainty of stimulants in time. The obtained results showed, that regardless of the age and income of saving people safety, the best choice, is to deposit their financial surpluses accordingly to the option, that the assumed use of Individual Retirement Account, and then after exceeding the yearly payment limit to deposit the capital in the taxed saving form. By the comparable interest rate it allows to get higher profit because it is partially free from taxation

Summarizing, it should be said that the long term profit level forecasting requires building and using the stochastic simulation model. Though the synthesis of the obtained simulation results has to be done with special caution and strictly refer to the accepted assumptions. It means that the conclusions

[10]The information effectiveness problem is described in por. [5], [6].

drawn on the basis of dependent variable trajectory analysis should be referred to the assumptions regarding both stimulants and reaction area.

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