Long-range dependencies in reading memory pages in the 
man-computer system interaction

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Abstract – This paper shows that the organization of computer system memory as a hierarchical 
structure can lead to the existence of long-memory effect during the man-computer interaction. As 
a point of reference, the rate at which the operating system resolves data from hard drive caused by 
the hardware page faults in the virtual memory mechanism can be given. The existence of such a 
phenomenon can be considered in terms of statistical mechanics and Hurst parameter.

1 Introduction

The modern computer systems consist of many interdependent subsystems. There 
are two possible approaches to their analysis. The first one is based on the idea of re-
ductionism proposed by Descrates[1] and can be considered as a still ruling paradigm in 
the case of computer science and engineering. The second one can be related to the still new idea of the complex systems approach, where in order to understand the behaviour 
of such systems one needs to have the knowledge about the behaviour of system com-
ponents and also, that is more important, how they act together [2, 3, 4, 5, 6, 7, 8].

This specific paradigm change can be even shown in the case of the idea of Turing 
machines and new approach for consideration in the case of interactive processing – 
even the opinion that “the computer engineering is not a mathematical science” was 
presented [9]. It should be noted that the Turing machine is a mathematical idea 
[10] while its implementations are the physical ones [11], but if the physical nature

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of computer systems was indicated, there is a need to have an appropriate physical (thermodynamical) basis for deliberations in such a case. In [12] it has been shown that the analysis of processes in the computer systems can be based on non-extensive thermodynamics. However, this analysis considers only spatial correlations, meanwhile in this paper we will focus on time dependencies leading to a long-memory effect.

The paper is divided into 5 sections. In Section 2 the conception of the hierarchical structure of memory system in the computer is presented. This proposal was given in [11] where its influence on computer thermodynamics was discussed. In Section 3 there is a short survey about long-range dependencies whereas Section 4 discusses practical usage of theoretical considerations. The last section concludes the paper.

## 2 Hierarchical structure of computer memory system

One of the most frequent properties of many complex systems is the hierarchy feature. This term can be related to the arrangement of elements, objects, grades, orders, people, values, classes, etc., in a graduated or ranked series [13]. Some parts (sub-systems) of the computer system also show such a structure which can be a natural consequence of some limitations. In the case of the computer memory system this is caused by physical (capacity, access time) and economical (cost per unit) conditions that result in a few levels (see Fig. 1).

![Hierarchical structure of memory subsystem](Fig. 1. Hierarchical structure of memory subsystem [11].)
In our study for the presented considerations we use the possibility of tracing some processes occurring in the system by the mechanisms that are normally implemented in operating systems – such a possibility gives a perfmon tool. This powerful program allows graphical visualization of many system counters behaviour and can be also used for data storage on hard drive [14]. For analysis only one counter was chosen, i.e. Memory\Page Reads/s because it reflects the rate at which the disk was read to resolve hard drive page faults. In other words, it is a very good indicator of processes that can occur in virtual memory: it shows the number of reading operations, without regard to the number of pages retrieved in each operation, as a main indicator of faults in the cache memory.

This counter behaviour was traced for many hours of computer system work, the time includes \( \approx 150 \cdot 10^3 \) observations with the resolution 3600 records per 1 hour. During the experiment the system was traced when the user used it for normal work – we did not process any special applications like benchmarks or other tests. This approach can be criticized because we did not use any scenarios in our experiment, as a consequence we can not repeat it, but on the other hand it should be noted that computer systems usually do not work in conditions that exhibit extreme workload caused for example by special programs or benchmarks. Computers are usually used for office applications, for entertainment (games, videos, music) or Internet exploration. Our approach ensures that the whole experiment and collected data reflect the real nature of the average user (computer session) and processes that can appear during processing in computer systems when it is used for personal work or entertainment.

The obtained time series of Memory\Page Reads/s counter are presented in Fig. 2. In some moments the time “picks” of high numbers of read operations can be visible. This feature is reflected in probability distribution – details can be found in [12]. But one can see also another interesting property, i.e., aggregation causing bursty behaviour: clustering of misses over some periods of time. Bursts are followed by gaps with no misses, in which a computer system (more precise CPU) requests data from hard drive – in the gap computer system computes the data obtained during the burst.

### 3 Long-range dependencies

Usually in the cases of long-range dependent statistical models the fractional Brownian motion (fBm) is considered. Mandelbrot and Van Ness proposed this model and it was associated with the Hurst parameter \( H \) as a natural expansion of Brownian motion. Mandelbrot and his followers showed that many natural phenomena can be characterized by fBm.

A process \( X(t) \) can be considered as a self-similar if for some \( H > 0 \) one has

\[
X(at) \approx a^H X(t) \quad \text{for} \quad a > 0.
\]

(1)

This can be interpreted as a situation in which the process is invariant under suitable translations of time and scale. If \( t \) is interpreted as a time and \( X(t) \) as a space, then
equation (1) says that every change of time scale \( a > 0 \) corresponds to the change of space scale \( a^H \). The \( H \) in fBm case shows the process memory effects and also gives information about persistence in time series [15]. The Gaussian processes are not the only ones that can be self-similar. It is known that there is a second class of such processes – the Lévy (\( \alpha \))-stable processes where \( H = 1/\alpha \). They can also have a memory effect described by \( d \) parameter and

\[
H = 1/\alpha + d. \tag{2}
\]

The self-similar processes became very important in modelling of many phenomena. For example, the existence of long-range dependencies was found in the computer networks. However, computer engineering lacks such a (statistical) analysis, that should possibly show influence of long-range dependencies on the computer systems or their subsystems behaviour. There are a few exceptions (see for example [5], where the problem of long-range dependencies in algorithmic computing was presented or [16] where the non-extensive statistical mechanics in insertion-sort was presented) but generally it is not even known how such processes influence the performance of memory, Central Processing Unit, operating systems, etc.

4 Results of investigations

The analysis in this section was done using on a set of data presented in Fig. 2. In order to ensure that the possible existence of long-memory effect is real – not an artifact – we follow a way ensuring reliable results. It should be noted that in many
cases similar analysis are done without some steps that seem to be necessary when one wants to have important results. Our approach will be presented in successive subsections.

### 4.1 Problem of stationarity

Any statistical analysis that considers the problem of possible existence of long-memory effects and self-similarity should be done for time series that are stationary. For many cases this feature is rather exception than a rule thus it is usually acceptable that the process exhibits weak-stationarity or asymptotical stationarity. In the presented analysis to solve this problem we use a method that is based on the quantile line test \[17\]. In this method it is assumed that the process is stationary when its probability distribution does not change over time and this fact is noted not by analysis of its mean and variance but by analysis of behaviour of quantiles. If computed quantiles do not change in time, i.e., they are parallel to the time axis, there is no evidence that time series are non-stationary. However, the usage of this method is limited by the fact that we need to have a set of time series in order to calculate quantile for each time lag. We did not have such a set but we follow different approaches, which is also acceptable \[17\] – we divided original time series into 40 subseries and considered them as copies of original set calculated quantiles. Because we have \(\approx 150 \cdot 10^3\) observations, this gave us 40 sets consisting of 3750 observations. However, such a number of time series did not guarantee that that results will be reliable (for each time lag quantiles will be calculated base on 40 records) thus for each calculated quantile we took a window of the width \(w = 30\) which guaranteed that for each calculated quartile there were used \(40 \cdot 30 = 1200\) records. The results of this method can be found in Fig. 3.

It can be noted that some of the calculated quantiles are not parallel to the time axis, however, on the other hand there is not a well visible trend (thus in the process a mean value does not change over time) or during time the distance between quantiles does not change. Taking into account this information and based on the results obtained in \[12\] where it was shown that the probability distribution of this process is well-defined (it also exhibits power-law) we assume that the process is stationary (at least it can be assumed that in this case we have asymptotical stationarity).

### 4.2 Existence of long-memory effect

The possible existence of long-memory effect was done based on seven different methods. Such an approach was taken because the calculations of \(d\) parameter (or equivalently Hurst \(H\) exponent) are not easy. The following methods were used:

- Absolute Value Method (AVM) – it calculates the \(H\) value,
- Aggregated Variance (AV) – it calculates the \(d\) parameter,
- R/S – it calculates the \(d\) value,
- Variance of Residuals (VR) – it calculates \(d\) exponent,
- Spectral density – it calculates \(d\) value,
- Whittle Estimator – it calculates the $d$ value,
- Wavelet Method – it calculates the $d$ value.

In the case of the AVM method the value of $d$ can be calculated from equation (2) in relation to the results obtained in [12] where it was indicated that $\alpha = 1.556$.

The first five methods are graphical. The sixth method is based on the maximum likelihood estimation for the periodogram (spectral density) and similarly to the last one they give only one value but with the confidence intervals. The results for the time series $X(t)$ are presented in Fig. 4 and in Tab. 1.

Table 1. List of all methods with the obtained values for the long-range memory effect measured by the $d$ parameter.

<table>
<thead>
<tr>
<th>Method</th>
<th>$d$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregated Variance</td>
<td>0.133</td>
</tr>
<tr>
<td>R/S</td>
<td>-0.034</td>
</tr>
<tr>
<td>Spectral density</td>
<td>0.334</td>
</tr>
<tr>
<td>Absolute Value Method</td>
<td>0.261</td>
</tr>
<tr>
<td>Variance of Residuals</td>
<td>0.706</td>
</tr>
<tr>
<td>Whittle Estimator</td>
<td>0.27</td>
</tr>
<tr>
<td>Abry-Veitch Estimator</td>
<td>0.342</td>
</tr>
</tbody>
</table>

As it can be seen only in the case of the R/S method we have no evidence that the analyzed time series shows the existence of long-memory effect, i.e. $d < 0$. This can
be caused by the fact that the method was prepared rather for the series that have probability distributions with the finite value of variance. As in [12] we show that the analyzed time series have power-law behaviour of distribution tails, the this can be an explanation. In the case of variance of residuals we have a situation in which obtained results are hard for interpretation. This can be a result of the following problem that can appear in this method. In the case of the processes where we have an infinite value of variance, the scatter of points used for $d$ estimation can be too large. However, such a situation does not follow from Fig. 4.

In methodology for estimation of long-memory effects it is sometimes very good to verify the obtained results to have conviction that they are relevant. This can be done based on bucket shuffling. This method is based on decoupling long-range from short-range dependencies in a series to check whether the existence of memory effect is an artifact or not. The time series are divided into buckets of length $b$ and then two levels of shuffling can be used. Internal shuffling assumes that the order of buckets is unchanged but the content of each bucket is shuffled – this breaks short-range dependencies. In external shuffling the content of each bucket is unchanged but the order of buckets is shuffled. To ensure that time series are well shuffled, it is good to use both methods.

In our case we shuffled the data by both methods: firstly by external shuffling with $b = 64$ and then by internal shuffling with $b = 32$. As a next step we calculated $d$ parameter with the use of all methods. The results are presented in Fig. 5 and Tab. 2.
Table 2. The list of all methods with the obtained values for the shuffled data (long-range memory effect is measured by the $d$ parameter).

<table>
<thead>
<tr>
<th>Method</th>
<th>$d$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregated Variance</td>
<td>0.006</td>
</tr>
<tr>
<td>R/S</td>
<td>-0.122</td>
</tr>
<tr>
<td>Spectral density</td>
<td>0.034</td>
</tr>
<tr>
<td>Absolute Value Method</td>
<td>0.094</td>
</tr>
<tr>
<td>Variance of Residuals</td>
<td>0.504</td>
</tr>
<tr>
<td>Whittle Estimator</td>
<td>0.164</td>
</tr>
<tr>
<td>Abry-Veitch Estimator</td>
<td>0.145</td>
</tr>
</tbody>
</table>

Fig. 5. Graphical representation of all used methods for the shuffled time series $X(t)$. Dotted line shows the linear fit obtained by the least mean square method. We have (from left to right, from top to bottom) 4a - the Aggregated Variance, 4b - R/S, 4c - the Spectral density, 4d - the Absolute Value Method, 4e - the Variance of Residuals.

Taking into account the results presented in Tab. 2, it can be seen that in most methods the parameter $d$ has lower values, closer to 0 indicating that in the shuffled data there is no memory effect. On the other hand, this also confirms the results obtained for the original time series $X(t)$.

5 Conclusions

In this paper we showed that in the case of man-computer interaction there is a strong evidence of long range memory effect. Taking into account the results obtained in [12],
it can be assumed that the interaction between man and computer system is governed by long range dependencies in the space and time domain. This conclusion is confirmed by special behaviour of probability distributions and by statistical properties of time series. Some weakness of our approach can be connected with the assumed conditions during experiment; it cannot be exactly repeated because we assume some “average” behaviour of the computer user. On the other hand, as justification, it should be taken into account that normally computer systems do not process extreme workload.

Acknowledgements

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