DOI:10.17951/h.2020.54.2.41-51

A N N A L E S
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN – POLONIA

VOL. LIV, 2 SECTIO H 2020

ŁUKASZ MARKOWSKI

lukasz.markowski@uni.lodz.pl
University of Lodz. Faculty of Economics and Sociology
3/5 Polskiej Organizacji Wojskowej St., 90-255 Lodz
ORCID ID: https://orcid.org/0000-0002-3921-1820

JAKUB KELLER

jakub.keller@uni.lodz.pl
University of Lodz. Faculty of Economics and Sociology
3/5 Polskiej Organizacji Wojskowej St., 90-255 Lodz
ORCID ID: https://orcid.org/0000-0002-8213-9887

Fear Anatomy – an Attempt to Assess the Impact of Selected Macroeconomic Variables on the Variability of the VIX S&P 500 Index

Keywords: VIX index; volatility index; macroeconomic indicators; market efficiency; U.S. Stock Exchange

JEL: G1; G14; G15

How to quote this paper: Markowski, Ł., & Keller, J. (2020). Fear Anatomy – an Attempt to Assess the Impact of Selected Macroeconomic Variables on the Variability of the VIX S&P 500 Index. *Annales Universitatis Mariae Curie-Skłodowska, sectio H – Oeconomia*, Vol. 54, No. 2.

Abstract

This article deals with the subject of volatility of financial markets in relation to the US stock market and its volatility index, i.e. the VIX index. The authors analyzed previous studies on the VIX index and based on them, defined a research gap that relates to the problem of market response to emerging macroeconomic information about the US economy. The vast majority of research on the VIX index relates to its forecasting based on mathematical models not taking into account current market data. The authors attempted to assess

the impact of emerging macro data on the variability of the VIX index, thus illustrating the magnitude of the impact of individual variables on the so-called US Stock Exchange fear index. The study analysed 80 macroeconomic variables in the period from January 2009 to June 2019 in order to check which of them cause the greatest market volatility. The study was based on correlation study and econometric modeling. The obtained results allowed to formulate conclusions indicating the most important macroeconomic parameters that affect the perception of the market by investors through the pricing of options valuation on the S&P 500 index. The authors managed to filter the most important variables for predicting the change of VIX level. In the eyes of the authors, the added value of the article is to indicate the relationship between macro variables and market volatility illustrated by the VIX index, which has not been explored in previous studies. The analyzes carried out are part of the research trend on market information efficiency and broaden knowledge in the area of capital investments.

Introduction

The VIX index was initially introduced by Whaley (1993). It was a new tool for investors to predict short-term market volatility by taking into account short term option prices for the S&P 100 index (OEX). Nowadays the VIX index is used as a benchmark for determining sensitivity of S&P 500 (SPX) market prices fluctuations in a 30-day timeframe. As mentioned, in the first place, the VIX was intended to predict the volatility of S&P 100 index. The reason for this was that the S&P 100 was more popular than S&P 500 in terms of investing activity at the early 1990s in the United States. The situation changed in September 2003 when CBOE introduced new methodology for the VIX calculations, considering the fact that the SPX option market became more popular than the OEX option market among investors. The newly introduced formula used in the VIX index calculation is:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

where:

σ – the value of the VIX divided by 100,

T – time to expiration of the option contract,

F – forward index level derived from option prices,

K_i – the strike price of the i^{th} out-of-the-money option (a call if $Kt > F$ and a put if $Kt < F$),

ΔK_i – the interval between strike prices,

r – the risk-free interest rate up to the expiration of the option contract,

$Q(K_i)$ – the midpoint of the bid-ask spread for each option with strike Ki.

The authors noticed that most of studies on the VIX are focused mainly on the mathematical-statistical approach, so the main intention was to propose more financial perspective on the market volatility depictured by the VIX. The aim of the article is to examine the impact of selected cyclically appearing macroeconomic data for the US market on the volatility of the VIX index. The main hypothesis

stated by the authors says that it is possible to select a set of variables that significantly increase the possibilities of forecasting VIX index volatility. The supportive hypotheses are:

H1: Variables regarding the labor market are particularly significant for investors' behavior in the context of the VIX index.

H2: Variables regarding the housing market are particularly significant for investors' behavior in the context of the VIX index.

H3: Variables built as a difference between the forecast and the actual value are more important to investors than variables defined as a difference between the past and current period.

Literature review

Most of the studies on VIX mechanics and predictions are based on the mathematical-statistical or stochastic approach as indicated by Chung, Tsai, Wang, Weng (2011), using the Heston model and encompassing regression to determine the information content of VIX on the SPX dynamics as well as by Mencia and Sentana (2013) who conducted empirical valuation on VIX derivative models. Similar approach was used by Aramian (2014) who focused on the Merton Jump Diffusion model. Lalancette and Simonato (2017) examined how a model articulated over a time-varying non-Gaussian distribution with conditional skewness and kurtosis can contribute to the overall explanation of the VIX dynamics. Goard and Mazur (2013) examined a series of continuous-time volatility models in terms of their ability to capture VIX behavior. Shu and Zhang (2012) examined the price-discovery function of the VIX and its information efficiency. Luo and Zhang (2012) studied the term structure of the VIX, establishing a new approach to catch information contained in it. Zhu and Lian (2012) developed an analytical "umbrella" formula for VIX futures and its applications. Bekaert and Hoerova (2014) analysed the squared VIX index, derived from US S&P 500 options prices, into the conditional variance of stock returns and the equity variance premium. Daigler, Dupoyet, and Petterson (2016) focused on the implied convexity of VIX futures, finding that the empirically estimated implied convexity can predict the future realized variance of VIX future prices. Huskaj and Nossman (2013) developed the term structure model for the VIX, determining VIX future prices exogenously. Another approach was presented by Huang, Shalistovich, Schlag, and Thimme (2018). They presented a volatility-of-volatility and showed that it is a separate risk factor affecting option returns. A study performed by Badshah, Frijns, and Tourani-Rad (2013) revealed a strong connection of VIX readings to exchange rate and gold index. Han, Kutan, and Ryu (2015) checked the impact of macroeconomic announcements using HAR models, and came to conclusion that they a have significant impact on the Korean volatility index (VOKSPI). Nikkinen and Vähämaa (2009) identified 16 US macroeconomic announcements that are

important in terms of FX implied volatility. Arshanapalli, d'Ouville, Fabozzi, and Switzer (2006) concluded the US bond markets increase on the day of announcements. Bomfim (2003) studied that unexpected US monetary policy announcements lead to a significant increase in market volatility. Jones, Lin, and Masih (2005) did the same for UK markets concluding the opposite effect. Srinivasan (2017) checked the impact of macroeconomic announcements on Indian volatility. Clements and Chen (2007) revealed that the VIX index falls on the day of Federal Open Market Committee meetings.

As the mathematical-statistical attempts to predict and determine the VIX are more and more popular, the authors tried to take a different approach. The issue contained in this paper is most closely related to the research performed by Onan, Salih and Yasar (2014). They examined the impact of macroeconomic announcements on the high-frequency behavior of the observed implied volatility skew of S&P 500 index options and the VIX. As a result, they concluded the VIX as a good measure for S&P 500 index volatility predictions. The authors observed a research gap in that field, and decided to take a broader look at the issue of the macroeconomic announcements' impact on VIX volatility.

Research methods

In this study, the authors attempted to examine the impact of selected macroeconomic variables from the US market on the level of the VIX market volatility index. The research sample covers a period of almost 10 years. The study period begins in January 2009 and ends in June 2019. The analysis applies to VIX index levels on a daily basis following the studies conducted by Marshall, Musayev, Pinto, and Tang (2012) and Fuss, Mager, Wohlenberg, and Zhao (2011).

The study included a set of macroeconomic parameters regarding the condition of the American economy, which are published cyclically. The criterion for selecting the variables studied was primarily their publication frequency, which should be at least once a month. This assumption was taken into account due to the need to obtain the appropriate number of observations for analysis. Most of the tested parameters appear more often – e.g. once a week.

Due to the large number of cyclical data that could be included in the study only because of the frequency of publication, an additional criterion for selecting variables was also adopted. This criterion assumes taking into account only those parameters that are perceived by investors as significant for the market. The significance of individual variables (according to investing.com and macronext.com) is described on a 3-point scale: insignificant (1), moderately significant (2) and very significant (3). In the analyzes, only parameters 2 or 3 were used. With the selection criteria described above, 40 macroeconomic variables were used in the final study.

Table 1. Macroeconomic variables included in the research

Investing.com Gold Index	U.S. Industrial Production MoM
U.S. Markit Composite Purchasing Managers Index (PMI)	U.S. Business Inventories MoM
U.S. Services Purchasing Managers Index (PMI)	U.S. TIC Net Long-Term Transactions
U.S. ISM Non-Manufacturing Purchasing Managers Index (PMI)	U.S. Building Permits
U.S. JOLTs Job Openings	U.S. Building Permits MoM
U.S. MBA 30-Year Mortgage Rate	U.S. Existing Home Sales MoM
U.S. Crude Oil Inventories	U.S. Existing Home Sales
U.S. Cushing Crude Oil Inventories	U.S. New Home Sales
U.S. 10-Year Note Auction	U.S. New Home Sales MoM
U.S. Initial Jobless Claims	U.S. Core Durable Goods Orders MoM
U.S. Continuing Jobless Claims	U.S. CB Consumer Confidence
U.S. Core Producer Price Index (PPI) MoM	U.S. Gross Domestic Product (GDP) QoQ
U.S. Baker Hughes Oil Rig Count	U.S. Goods Trade Balance
U.S. Federal Budget Balance	U.S. Pending Home Sales MoM
U.S. Export Price Index MoM	U.S. Core PCE Price Index MoM
U.S. Core Retail Sales MoM	U.S. Chicago Purchasing Managers Index (PMI)
U.S. Nonfarm Productivity QoQ	U.S. ISM Manufacturing Purchasing Managers Index (PMI)
U.S. Philadelphia Fed Manufacturing Index	U.S. ADP Nonfarm Employment Change
U.S. NY Empire State Manufacturing Index	U.S. Nonfarm Payrolls
U.S. Retail Sales MoM	U.S. Unemployment Rate

Source: Authors' own study.

The variables presented above are published cyclically. The way the data is presented allows them to be included in a survey in a variety of ways. First of all, it should be noted that in addition to information about the previous and current level of a given indicator, the value that was predicted by the analysts for the given period is also disclosed. For this reason, it is possible to analyze the change in individual indicators from period to period and their further interpretation indicating whether, in relation to a given indicator, the economic situation improves or worsens. In addition, due to published analysts' forecasts for individual indicators, it is also possible to construct a variable indicating the convergence of actual data with earlier forecasts.

According to the authors, market participants discuss analysts' predictions by making appropriate transactions before publishing actual data for a given period. If the projections significantly differ from the real data, we should be dealing with an increased activity of investors, which is caused by the need to revise investment positions taken. Obviously, an additional activity of investors should contribute to higher levels of the VIX index at a given moment (at the trading session during which the new data was published).

At this stage of the analysis, the authors formulated a supporting hypothesis that the variables describing the differences between analysts' predictions and real data

will have a greater impact on the VIX index than the variables describing the change of a given parameter between the current and previous reading.

The variables finally used in the study were defined as:

$$Xj_{ch} = X_{ji} - X_{ji-1}$$

where:
Xj_{ch} – change of parameter between current and previous reading
X_{ji} – current reading of parameter j
X_{ji-1} – previous reading of parameter j

$$Xj_{for} = X_{ji} - X_{jfi}$$

where:
Xj_{for} – difference between forecast of parameter j and the actual reading
X_{jfi} – forecasted value of parameter j for period i

The authors examined the correlation relationship between defined variables and the VIX index level. Because two variables were defined for each macroeconomic variable: as a change from period to period and as a difference from forecasts, the total number of variables tested was 80. The study checked the correlations of all defined variables and their significance. The strongest and statistically significant ones were selected from the tested compounds.

In the further part of the study, there was performed the analysis of the impact of variables most strongly correlated with the VIX index using the probability model. The aim of this part was to test the repeatability of the impact of the studied variables on the analyzed index. This part of the analysis indicates whether the examined variables are likely to cause sufficiently large changes in the level of the VIX index relative to the adopted historical reference point.

The model has been defined in such a way that with its help it is possible to assess the chances of causing the change of the size of the VIX index beyond the range determined by the average value of the index from the previous five trading sessions that is assumed to be caused by the publishing of the previously selected variables.

In the case of undertaken analyzes, this variable was determined as follows:

$$y_i = \begin{cases} 0, if\ VIX_i \in \{mean(VIX_{i-5}) - \delta(VIX); mean(VIX_{i-5}) + \delta(VIX)\} \\ 1, \quad \text{in other cases} \end{cases}$$

probability studies are based on the logit model.

Based on the calculated correlation relationships and on the basis of the conclusions drawn from the logit models, the authors drew conclusions regarding the research hypotheses. Correlation analysis was set to filter the significant variables

from the studied pool and the logit modelling helped to determine the analysis of the repeatability of the strong influence of individual variables on the VIX index.

Results

As part of the study, the authors first examined the correlation between the VIX index level and proposed variables. Using the Pearson correlation test allowed to create a hierarchical list of variables indicating those parameters that are most closely related to the examined index and those that show very low association with it. It should be emphasized that parameters that showed a high level of correlation were also noted, which, however, was not confirmed by the statistical significance of this relationship. In this case, the authors excluded the variable from the ranking. Based on this analysis and selection, a list of variables most strongly related to the VIX level was obtained (Table 2).

Table 2. Variables correlated the most with the VIX index

Variable	Symbol	Correlation	Significance
U.S. Unemployment Rate	X40ch	0.56	***
U.S. Core Producer Price Index (PPI) MoM	X12for	0.24	***
U.S. Existing Home Sales MoM	X26ch	0.22	***
U.S. Existing Home Sales MoM	X26for	0.21	**
U.S. Gross Domestic Product (GDP) QoQ	X32for	0.20	***
U.S. Business Inventories MoM	X22ch	-0.19	**
U.S. ISM Manufacturing Purchasing Managers Index (PMI)	X37ch	-0.30	***
U.S. Core PCE Price Index MoM	X35for	-0.31	***
U.S. ISM Manufacturing Purchasing Managers Index (PMI)	X37for	-0.34	***
U.S. Business Inventories MoM	X22for	-0.35	***

Note: ** 5% level of error, *** 1% level of error

Source: Authors' own study.

A set of variables with the lowest level of association with the VIX was also selected similarly. The list of these parameters is presented in Table 3.

Table 3. Variables correlated the least with the VIX index

Variable	Symbol	Correlation	Significance
U.S. Unemployment Rate	X40for	0.04	**
U.S. Initial Jobless Claims	X10for	0.03	***
U.S. Crude Oil Inventories	X7for	0.02	***
U.S. Continuing Jobless Claims	X11for	0.02	***
U.S. Chicago Purchasing Managers Index (PMI)	X36ch	0.00	***
U.S. ISM Non-Manufacturing Purchasing Managers Index (PMI)	X4ch	-0.01	**
U.S. Industrial Production MoM	X21for	-0.02	***
U.S. Chicago Purchasing Managers Index (PMI)	X36for	-0.03	***
U.S. Philadelphia Fed Manufacturing Index	X18ch	-0.04	**
U.S. ISM Non-Manufacturing Purchasing Managers Index (PMI)	X4for	-0.04	***

Note: ** 5% level of error, *** 1% level of error

Source: Authors' own study.

It should be noted that despite the initial selection of data, which according to surveys conducted among investors are at a similar level of impact on the VIX index, the results obtained show significant discrepancies in market responses to information contained in individual variables.

In order to extend the conclusions of the variable correlations, the authors also modelled the probability of a significant change in the VIX index level in relation to the previously defined expectation. The results of calculations using the logit model for individual variables from Table 2 are presented in Table 4.

Table 4. Results of logit modeling of the VIX with preselected variables

Model	Variables included	Coefficient	z-score	Percentage of correct predictions
1	X40ch	373.64	1.2209	69.20%
2	X12ch	-139.63	-1.4775	59.20%
3	X26ch	-10.10	-1.1801	–
	X26for	7.40	0.5018	59.00%
4	X32for	-21.46	-0.2970	51.10%
5	X22ch	-209.04	-2.2399	–
	X22for	335.65	1.9884	61.10%
6	X37ch	-0.14	-0.1885	–
	X37for	0.30	0.3327	61.90%
7	X35for	355.86	0.5808	61.50%

Source: Authors' own study.

The econometric estimations carried out again show discrepancies in the impact of individual variables on the examined index. The highest probability of a significant

change in the VIX level is caused by the change in "U.S. Unemployment Rate", while the smallest is related to the variable based on the error of analysts' forecasts within the scope of "U.S. Gross Domestic Product (GDP) QoQ".

Discussion

The study allowed to draw interesting conclusions regarding the impact of individual macroeconomic variables on the VIX index. On the basis of statistical analysis, the authors showed significant differences in the impact of the tested parameters on changes in indexes at individual sessions. The study revealed a considerable impact of unemployment data readings on analysts' opinions. It is visible both in terms of collinearity of changes and their dynamics, which is demonstrated by the results of the logit model.

The vast majority of publications about the VIX index are based on the mathematical approach, leaving behind the impact of economic announcements as an index-shaping factor. Taking into account the research gap, the studies performed in this paper gave promising results on identifying significant signals from the market, impacting the VIX for S&P 500 index. The research conducted by Onan, Salih, and Yasar (2014) gave similar conclusions.

The main purpose of the presented study was achieved and the main hypothesis, stating that there is a certain set of specific variables with a relatively strong impact on the VIX index, has been positively verified. Also supporting hypotheses H1 and H2 have been positively verified, which means that the variables regarding labor and housing market are significant in shaping the index level. However, the supportive hypothesis H3, indicating that variables based on the difference between analysts' predictions and the real reading of individual variables, would have a stronger impact on the VIX index, has been verified negatively.

It is a surprise for the authors that the variables based on the change from period to period constituted an equally strong premise for the corresponding increases or decreases in the level of the VIX index, as was the case with variables of the "Xfor" type defined in the study. It should be noted that in many cases, analysts' predictions largely coincide with actual readings of individual parameters. Therefore, it should be expected that investors will take the relevant investment positions related to given projections in advance. The study, however, showed that many seemingly significant parameters were not clearly related to VIX quotations. This fact is also an indication to say that the rankings of variable significance presented on investment sites have a subjective dimension and do not necessarily reflect the real impact on investors.

Conclusions

The purpose of this article was to verify the impact of individual macroeconomic variables on the VIX market volatility index. The obtained results confirmed the main hypothesis regarding a different impact of variables with a declared, similar level of impact on the market. Also, two out of three supporting hypotheses, regarding the labor market and production condition, have been verified as positive. The correlation study allowed the selection of the most significant variables out of the 80 selected parameters. The analysis, using probability models, led to similar conclusions, indicating significant discrepancies in the impact of the studied parameters on the examined index.

The authors are aware of the imperfections of the analysis that should be developed in subsequent studies, for example, by including further economic variables. The possibility of redefining the binary variable in logit modeling should also be considered. There is a wide variety of approaches to determine the reference value (expected value in the model). What is more, the authors would like to improve the accuracy of research by changing the data intervals to intraday in order to verify whether the conclusions obtained for daily rates of return are also similar in shorter periods of time, i.e. 15 min, 30 min and 1 hour after the macroeconomic data occurs (similar to Nofsinger & Prucyk, 2003).

However, the authors believe that the proposed study increases the added value due to the knowledge about decomposing the VIX index which in the context of current research is not a sufficiently exposed issue. The conclusions drawn from the inference have both scientific and practical importance, because the VIX index can be an element of building up investment strategies, which means that the nature of its variability is of interest to market practitioners.

References

Aramian, F. (2014). *Modeling VIX Futures and Pricing VIX Options in the Jump Diffusion Modeling*. Stockholm: Stockholm University.

Arshanapalli, B., d'Ouville, E., Fabozzi, F., & Switzer, L. (2006). Macroeconomic news effects on conditional volatilities in the bond and stock markets. *Applied Financial Economics*, *16*, 377–384. **doi:10.1080/09603100500511068**

Badshah, I.U., Frijns, B., & Tourani-Rad, A. (2013). Contemporaneous spill-over among equity, gold, and exchange rate implied volatility indices. *Journal of Futures Markets*, *33*(6), 555–572. **doi:10.1002/fut.21600**

Bekaert, G., & Hoerova, M. (2014). The VIX, the variance premium and stock market volatility. *Journal of Econometrics*, *183*(2), 181–192. **doi:10.1016/j.jeconom.2014.05.008**

Bomfim, A.N. (2003). Pre-announcement effects, news effects, and volatility: Monetary policy and the stock market. *Journal of Banking and Finance*, *27*, 133–151. **doi:10.1016/S0378-4266(01)00211-4**

Chung, S., Tsai, W., Wang, Y., & Weng, P. (2011). The information content of the S&P 500 index and VIX options on the dynamics of the S&P 500 index. *The Journal of Futures Markets*, *31*(12), 1170–1201. **doi:10.1002/fut.20532**

Clements, A., & Chen, E.T. (2007). S&P 500 implied volatility and monetary policy announcements. *Finance Research Letters*, *4*(4), 227–232. **doi:10.1016/j.frl.2007.07.002**

Daigler, R.T., Dupoyet, B., & Petterson, F.M. (2016). The implied convexity of VIX futures. *The Journal of Derivatives*, *23*(3), 73–90. **doi:10.3905/jod.2016.23.3.073**

Fuss, R., Mager, F., Wohlenberg, H., & Zhao, L. (2011). The impact of macroeconomic announcements on implied volatility. *Applied Financial Economics*, *21*(21), 1571–1580. **doi:10.1080/09603107.2011.583216**

Goard, J., & Mazur, M. (2013). Stochastic volatility models and the pricing of VIX options. *Mathematical Finance*, *23*(3), 439–458. **doi:10.1111/j.1467-9965.2011.00506.x**

Han, H., Kutan, A.M., & Ryu, D. (2015). Modeling and predicting the market volatility index: The case of VKOSPI. *Economics Discussion Papers*, *2015-7*.

Huang, D., Shalistovich, I., Schlag, C., & Thimme, J. (2018). Volatility-of-volatility risk. *Journal of Financial and Quantitative Analysis*, *54*(6), 2423–2452. **doi:10.1017/S0022109018001436**

Huskaj, B., & Nossman, M. (2013). A term structure model for VIX futures. *The Journal of Futures Markets*, *33*(5), 421–442. **doi:10.1002/fut.21550**

Jones, B., Lin, C., & Masih, M.M. (2005). Macroeconomic announcements, volatility, and interrelationships: An examination of the UK interest rate and equity markets. *International Review of Financial Analysis*, *14*, 356–375. **doi:10.1016/j.irfa.2004.10.001**

Lalancette, S., & Simonato, J. (2017). The role of the conditional skewness and kurtosis in VIX index valuation. *European Financial Management*, *23*(2), 325–354. **doi:10.1111/eufm.12096**

Luo, X., & Zhang, J. (2012). The term structure of VIX. *The Journal of Futures Markets*, *32*(12), 1092–1123. **doi:10.1002/fut.21572**

Marshall, A., Musayev, T., Pinto, H., & Tang, L. (2012). Impact of news announcements on the foreign exchange implied volatility. *Journal of International Financial Markets Institutions and Money*, *22*(4), 719–737. **doi:10.1016/j.intfin.2012.04.006**

Mencia, J., & Sentana, E. (2013). Valuation of VIX derivatives. *Journal of Financial Economics*, *108*(2), 367–391. **doi:10.1016/j.jfineco.2012.12.003**

Nikkinen, J., &Vähämaa, S. (2009). Central bank interventions and implied exchange rate correlations. *Journal of Empirical Finance*, *16*(5), 862–873. **doi:10.1016/j.jempfin.2009.05.002**

Nofsinger, J., & Prucyk B. (2003). Option volume and volatility response to scheduled economic new releases. *The Journal of Futures Markets*, *23*(4), 315–345. **doi:10.1002/fut.10064**

Onan, M., Salih, A., & Yasar, B. (2014). Impact of macroeconomic announcements on implied volatility slope of SPX options and VIX. *Finance Research Letters*, *11*, 454–462. **doi:10.1016/j.frl.2014.07.006**

Shu, J., & Zhang, J. (2012). Causality in the VIX futures markets. *The Journal of Futures Markets*, *32*(1), 24–46. **doi:10.1002/fut.20506**

Srinivasan, P. (2017). Macroeconomic information and the implied volatility: Evidence from India VIX. *Theoretical Economics Letters*, *7*, 490–501. **doi:10.4236/tel.2017.73037**

Whaley, R. (1993). Derivatives on Market Volatility: Hedging Tools Long Overdue. *The Journal of Derivatives*, *1*(1), 74–84. **doi:10.3905/jod.1993.407868**

Zhu, S., & Lian, G. (2012). An analytical formula for VIX futures and its applications. *The Journal of Futures Markets*, *32*(2), 166–190. **doi:10.1002/fut.20512**